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Some aspects of the approach to modelling assignments of the means for the tasks in municipal services companies

O pewnym podejściu do modelowania przydziału środków do zadań w przedsiębiorstwach komunalnych

Streszczenie: W artykule przedstawiono zagadnienie przydziału pojazdów do zadań w przedsiębiorstwach komunalnych w kontekście wyznaczania minimalnych tras jazdy pojazdów. Opracowano model matematyczny zagadnienia przydziału oraz zaproponowano metodę rozwiązującą zagadnienie przydziału w przedsiębiorstwach komunalnych. Metoda składa się z dwóch etapów, tj. etapu wyznaczającego minimalną trasę składającą się ze wszystkich zadań oraz etapu wyznaczania tras indywidualnych dla poszczególnych pojazdów. W świetle przeprowadzonych rozważań wskazując trasy indywidualne wyznaczymy zadania do realizacji, co jest tożsame z rozwiązaniem problemu przydziału. Zaproponowano algorytm genetyczny do rozwiązania problemu optymalizacyjnego przedstawionego w pierwszym etapie metody. Weryfikacja algorytmu potwierdziła jego skuteczność.

Słowa kluczowe: przedsiębiorstwo komunalne, przydział, optymalizacja

Abstract: In this article the assignment problem of vehicles to tasks in municipal services companies in the context of designating the minimum routes of vehicles was presented. The mathematical model of the assignment problem was developed, and proposed a method for solving the assignment problem in municipal services companies. The method consists of two stages i.e. the stage of designating the minimum route consisting of all tasks and the stage of designating individual routes for each vehicle. In the light of considerations indicating individual routes we designate the tasks to the implementation, which is the equivalent of solving the assignment problem. The genetic algorithm for solving the optimization problem presented in the first stage of the method was proposed. Verification of this algorithm confirmed its effectiveness.

Keywords: the municipal services companies, the assignment, optimization

Introduction

The main task of municipal services companies is waste collection from the region. The issue of waste collection is a complex decision which relates to the travelling salesman problem[1]. One of the main problems in the municipal ser-

[1] Płaczek E., Szołtysek J., *Wybrane metody optymalizacji systemu transportu odpadów komunalnych w Katowicach,* LogForum, Wyższa Szkoła Logistyki, Vol. 4, Poznań 2008, p. 1-10.

vices companies is vehicle routing[2]. This issue is difficult to solve because of the many points of waste collection, the limit of the working time of a driver, the limit of the driving time of a driver, ands the limit of the time for tasks realisation.

In order to solve the vehicles routing problem the authors of this publication presented the method which solves the issue of assignment of the means for tasks in municipal services companies. Designating the assignment problem presented in this paper is equivalent to designating the vehicles routing problem. The means we can define as the vehicles in the company. Defining the tasks is very difficult. Designating the tasks relies on indicating the point of the beginning of the task (on the Fig.1 the points a) and the point of completing the task (on the Fig.1 the points b). The point of ending we can define as the point where the vehicle leaves the route and goes to the dumping ground. The vehicle leaves the route of loading when the payload of the vehicle is exceeded. The points of beginning and ending start and end the route of the waste collection realized by the vehicles. Complexity of designating the tasks relies on that each point of loading is located in a different place. The task is designated when we know the routes of the vehicles which collect the waste (on the Fig.1 the blue arrows), i.e. the point of the beginning of the route, the points of the route and the point of finishing the route. Designating the route between point a and point b is a complex optimization problem.

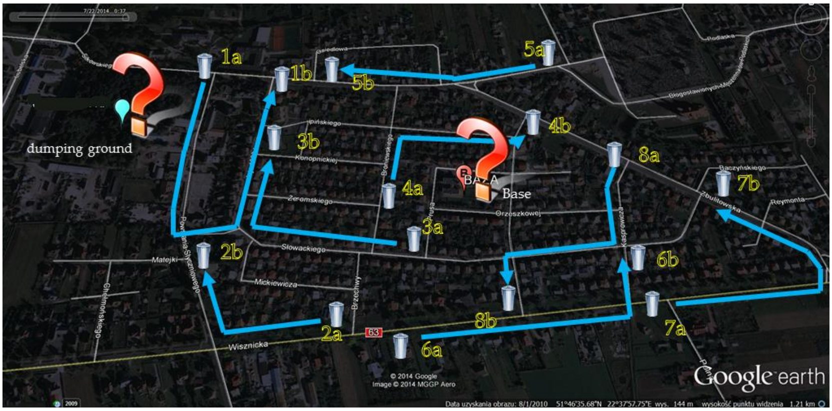


Fig. 1. The assignment problem in the municipal services companies
Source: compiled by author

The process of designating the tasks is not presented in this paper. It is assumed that all tasks are already well-known.

When all the tasks are known we can go on to assign the means for the tasks. The problem of assigning vehicles to tasks in municipal services compa-

[2] Beliën J., Boeck L., *Municipal Solid Waste Collection and Management Problems: A Literature Review*, Transportation Science, Institute for Operations Research and the Management Sciences (INFORMS) Volume 48 Issue 1, USA 2014, p. 78-102.
Bautista J., Fernández E., Pereira J., *Solving an urban waste collection problem using ants heuristics*, Computers & Operations Research, Elsevier, Volume 35, Issue 9, USA 2008, p. 3020-3033.

nies appears when the vehicle starts the first task and leaves the base. We can ask the question, which task will be realized as the first in the route (the question mark in Fig.1 near the base)? The problem of the assignment appears again when the vehicle has ended the current task, unloaded the waste on the dumping ground and we can ask the question again which next task will be realized on the route (the question mark in Fig.1 near the dumping ground). The assumption is that all tasks must be realized in the minimum route in a working day. We have three points of reference: the base, the task and the dumping ground which are the elements of the transport system. These points determine the routes of the vehicles. For the need of modelling the assignment it is assumed that the dumping ground is defined as the finishing point of the task. The task can be reached in two ways, directly from the dumping ground or through the base. The main aim of the assignment is to designate the next task on the route and the minimum route between two tasks for each vehicle. One should remember that there is a limit to the working time of a driver, a limit to the driving time of a driver, a limit of time for all tasks realization which impedes the performance of the assignment.

The main aim of the proposed method is to designate a set of tasks for the implementation of individual vehicles. The method consists of two stages. The first stage is to indicate the minimum route implementing all tasks. The second stage is to designate individual routes for each vehicle, assuming that the working time, driving time of the driver and time of task realization on these routes can't exceed permissible limitations. After designating the individual routes we know the tasks for implementation, we know the order of implementing these tasks, so we can assume that the assignment problem described above was solved. The individual routes are component parts of the minimum route thus they are the minimum routes. Designating assignments according to these routes is equivalent to designating the optimal assignment.

The authors of this paper concentrated mainly on the first stage of the presented method because this stage is the optimization stage and decides the further results. It described the genetic algorithm designating the minimum route which consists of all tasks and verification of the algorithm was presented.

The second stage was described only in a theoretical way, indicating the further steps of the presented assignment problem.

The assignment problem in the literature

In text sources the idea of the assignment problem is explained as the assignment of the established number of tasks to the specified number of agents on the assumption that the agents are able to perform these tasks. Each task can be assigned to exactly one agent and vice versa – each agent can perform only one task[3]. The assignment problem is a linear – discrete optimization problem. The variables take the value 1 or 0. The most popular method for solving

[3] Lonc Z., *Wstęp do algorytmicznej teorii grafów*, Centrum studiów zaawansowanych Politechniki Warszawskiej, Warszawa 2010, p. 75.

this problem is widely described in the literature as the Hungarian method[4]. There are methods which are based on the heuristic algorithms e.g. the genetic algorithms[5].

In real situations the assignment problem is modified and more difficult. This problem often appears in transport companies where many factors such as the number of vehicles and drivers, the limit of the driving time of a driver, the limit of the working time of a driver, the limit of realization of all tasks, the limit of payload of the vehicle, increase the complexity of the problem. In this case the methods of a linear - discrete optimization are ineffective. We can solve the problem using the effective heuristics methods e.g. the genetic algorithms[6], the ant algorithms[7], the particle swarm optimization[8]. The diversity of limits in the companies means that the problem of allocation of the means to assignment of tasks can be modified.

The mathematical model of the assignment of vehicles to the tasks

In order to present a mathematical model of the assignment the following data has been specified[9]:

- $\boldsymbol{W}^{\boldsymbol{Zp}} = \{1,...,i,...,\overline{W^{zp}}\}$ – the set of tasks,
- $\boldsymbol{W}^{\boldsymbol{Zk}} = \{1,...,j,...,\overline{W^{zk}}\}$ – the set of points of loading, the points where the vehicle leaves the route,
- $\boldsymbol{W}^{\boldsymbol{B}} = \{1,...,b,...,\overline{W^{b}}\}$ – the set of the numbers of bases,
- $\boldsymbol{W}^{\boldsymbol{W}} = \{1,...,k,...,\overline{W^{w}}\}$ – the set of points of unloading (e.g. the dumping ground),
- $\boldsymbol{W}^{\boldsymbol{P}} = \{1,...,p,...,\overline{W^{p}}\}$ – the set of vehicles' numbers,
- $\boldsymbol{W}^{\boldsymbol{N}} = \{1,...,n,...,\overline{W^{n}}\}$ – the set of drivers' numbers
- $\mathbf{W} = [w(k,i)]$ – the matrix of the distance between k-this point of unloading and i-this task, $\mathbf{BZ} = [bz(b,i)]$ – the matrix of the distance between b-this base and i-this task, $\mathbf{WB} = [wb(k,b)]$ – the matrix of the distance between k-

[4] Burkrd R., Dell'Amico M., Marttelo S., *Assignment problems*, Society for Industrial and Applied Mathematics. Philadelphia 2009, p. 79-87.
[5] Akbulut M., Yilmaz A., *A modified genetic algorithm for the generalized assignment problem*, Journal of electrical and electronics engineering, Istanbul 2009, pp. 951-958.
[6] Izdebski M., Jacyna M., *Some Aspects of the Application of Genetic Algorithm for Solving the Assignment Problem of Tasks to Resources in a Transport Company*, Logistic and Transport, Vol. 21, No 1 (2014), p. 13-20.
[7] Izdebski M., Jacyna M., *The algorithm solving the problem of allocation of tasks to resources in the transport company*, Conference Proceedings Carpathian Logistics Congress, December 9th - 11th 2013, Cracow, Poland, EU. (CD-ROM)
[8] Yusoff M., Ariffin J., Mohamed A.,*Solving Vehicle Assignment Problem Using Evolutionary Computation*, Lecture Notes in Computer Science Volume 6145, 2010 r., p. 523-532.
[9] Jacyna M., *Modelowanie i ocena systemów transportowych*, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa 2009.

this point of unloading and b-this base, $\mathbf{ZW} = [zw(j,k)] -$ the matrix of the distance between j-this point where the vehicle leaves the route and k-this point of unloading,

- $\mathbf{T1} = [t1(p,n,j,k)] -$ the matrix of travel times between j-this point where the vehicle leaves the route and k-this point of unloading for p-this vehicle and n-this driver, $\mathbf{T2} = [t2(p,n,b,i)] -$ the matrix of travel times between b-this base and i-this task for p-this vehicle and n-this driver, $\mathbf{T3} = [t3(p,n,k,b)] -$ the matrix of travel times between k-this point of unloading and b-this base for p-this vehicle and n-this driver, $\mathbf{T4} = [t4(p,n,k,i)] -$ the matrix of travel times between k-this point of unloading and i-this task for p-this vehicle and n-this driver, $\mathbf{T5} = [t5(p,n,k)] -$ the matrix of times of unloading a vehicle in k-this point of unloading for p-this vehicle and n-this driver, $\mathbf{T6} = [t6(p,n,i)] -$ the matrix of times of loading a vehicle in i-this task for p-this vehicle and n-this driver, $\mathbf{T7} = [t7(p,n,k)] -$ the matrix of waiting time for unloading in k-this point of unloading for p-this vehicle and n-this driver, $\mathbf{T8} = [t8(p,n,i)] -$ the matrix of driving time in i-this task for p-this vehicle and n-this driver,
- $\varphi(p) -$ payload p- this of vehicle, $\theta(i) -$ the volume of the task,
- T^{rest} – statutory resting time on the route, T^{dop1} – the permitted driving time, T^{dop} – the permitted working time of the driver, $\Delta T -$ the range of realization of the all tasks.

The main task is finding the following decision variables:

- $\mathbf{XBZ} = [xbz(b,i)]$, $xbz(b,i) \in \{0,1\}$, if $xbz(b,i) = 1$ then the route between b-this base and i-this task is used, otherwise $xbz(b,i) = 0$;
- $\mathbf{X} = [x(k,i)]$, $x(k,i) \in \{0,1\}$, if $x(k,i) = 1$ then route between k-this point of unloading and i-this task is used, otherwise $x(k,i) = 0$;
- $\mathbf{XWB} = [xwb(k,b)]$, $xwb(k,b) \in \{0,1\}$, if $xwb(k,b) = 1$ then the route between k-this point of unloading and b-this base is used, otherwise $xwb(k,b) = 0$;
- $\mathbf{XZW} = [xzw(j,k)]$, $xzw(j,k) \in \{0,1\}$, an auxiliary variable defining the route between the point where the vehicle leaves the route and the point of unloading, if $xzw(j,k) = 1$ then the route between j-this point and k-this point of unloading is used, otherwise $xzw(j,k) = 0$;

to the function of the criterion determining the assignment saved by the formulation:

$$F(\mathbf{XBZ}, \mathbf{X}, \mathbf{XWB}, \mathbf{XZW}) =$$
$$\sum_{b \in \boldsymbol{W}^{\boldsymbol{B}}} \sum_{i \in \boldsymbol{W}^{\boldsymbol{Zp}}} xbz(b,i) \cdot bz(b,i) + \sum_{k \in \boldsymbol{W}^{\boldsymbol{W}}} \sum_{i \in \boldsymbol{W}^{\boldsymbol{Zp}}} x(k,i) \cdot w(k,i) +$$
$$+ \sum_{k \in \boldsymbol{W}^{\boldsymbol{W}}} \sum_{b \in \boldsymbol{W}^{\boldsymbol{B}}} xwb(k,b) \cdot wb(k,b) + \sum_{j \in \boldsymbol{W}^{\boldsymbol{Zk}}} \sum_{k \in \boldsymbol{W}^{\boldsymbol{W}}} xzw(j,k) \cdot zw(j,k) \to \min \tag{1}$$

will take the minimum value.

The function of the criterion minimizes the total route consisting of all tasks.

Constraints take the form of:

- a limit to the driving time of a driver i.e. the sum of travel times from the base to the task and directly from the point of unloading, the sum of travel times between the point where the vehicle leaves the route and the point of unloading and the sum of travel times between the point of unloading and the base:

$$\forall n \in \boldsymbol{N}, \forall p \in \boldsymbol{P}$$
$$\sum_{b \in \mathbf{W}^{\mathbf{B}}} \sum_{i \in \mathbf{W}^{\mathbf{Zp}}} xbz(b,i) \cdot \left[2(p,n,b,i) + t8(p,n,i) \right] + \sum_{k \in \mathbf{W}^{\mathbf{W}}} \sum_{i \in \mathbf{W}^{\mathbf{Zp}}} x(k,i) \cdot \left[4(p,n,b,i) + t8(p,n,i) \right]$$
$$+ \sum_{k \in \mathbf{W}^{\mathbf{W}}} \sum_{b \in \mathbf{W}^{\mathbf{B}}} xwb(k,b) \cdot t3(p,n,k,b) + \sum_{j \in \boldsymbol{W}^{\boldsymbol{Zk}}} \sum_{k \in \boldsymbol{W}^{\boldsymbol{W}}} xzw(j,k) \cdot t1(p,n,j,k) \leq T^{dop1} \tag{2}$$

- a limit to the working time of a driver i.e. the driving time of a driver, the sum of loading times in the tasks, the sum of unloading times, the sum of expected times of unloading and resting time on the route, in order to reduce the left part of the formula (2) the driving time of a driver takes the mark T^{drive}:

$$\forall n \in \boldsymbol{N}, \forall p \in \boldsymbol{P}$$
$$T^{drive} + \sum_{b \in \mathbf{W}^{\mathbf{B}}} \sum_{i \in \mathbf{W}^{\mathbf{Zp}}} \sum_{za \in \mathbf{W}^{\mathbf{Zad}}} xbz(b,i) \cdot t6(p,n,i) + \sum_{k \in \mathbf{W}^{\mathbf{W}}} \sum_{i \in \mathbf{W}^{\mathbf{Zp}}} \sum_{za \in \mathbf{W}^{\mathbf{Zad}}} x(k,i) \cdot t6(p,n,i)$$
$$+ \sum_{j \in \boldsymbol{W}^{\boldsymbol{Zk}}} \sum_{k \in \boldsymbol{W}^{\boldsymbol{W}}} xzw(j,k) \cdot t5(p,n,k) + \sum_{j \in \boldsymbol{W}^{\boldsymbol{Zk}}} \sum_{k \in \boldsymbol{W}^{\boldsymbol{W}}} xzw(j,k) \cdot t7(p,n,k) + T^{rest} \leq T^{dop} \tag{3}$$

- the limit of realization of all tasks i.e. the tasks must be accomplished within a given range of time, in order to reduce the left part of the formula (3) the working time of a driver takes the mark T^{work}:

$$T^{work} \leq \Delta T \tag{4}$$

- the limit of payload of the vehicle i.e. the vehicle can realize the task if the payload of this vehicle is greater or equal to the size of this task:

$$\forall i \in \boldsymbol{W}^{\boldsymbol{Zp}}, \forall p \in \mathbf{P}$$
$$\theta(i) \leq \varphi(p) \tag{5}$$

The method of designating the assignment vehicles to tasks

The method of designating the assignment is described in two stages. In the first stage the minimum route which consists of all tasks is designated.

The measure of the presented assignment is the length of the routes of vehicles implementing assigned tasks. The minimum route which consists of all tasks will designate the minimum lengths between the tasks and indicate tasks in the route of each vehicle. On the basis on this route the individual routes for each vehicle are designated, which is unambiguously with solving the assignment problem presented in this paper.

The main task of the second stage of the presented method is just to designate the individual routes for each vehicle. In order to realize the second stage we must introduce the concept of the elementary route. The elementary route is part of the minimum route which is designated in the first stage. The minimum route consists of two kinds of elements i.e. the task and the base. The beginning and the ending of each elementary route is always the base. For each route all limits i.e. working time, driving time of the driver and the time of tasks realization are checked. The individual routes for each vehicle are designated by combining elementary routes. If two elementary routes fulfil the time limits we can combine another route. Otherwise we start creating a new individual route for another vehicle.

The minimum route was designated by the genetic algorithm, which is used as a functional and practical optimization tool[10].

The genetic algorithm consists of the following steps: Step 1 – The determining of a structure of the data processed by the algorithm, Step 2 – The determining of the function of adaptation, Step 3 – The selection of chromosomes dependent on the function of adaptation, Step 4 – The crossover of chromosomes, Step 5 – The mutation of chromosomes, Step 6 – The inversion.

The steps from three to six of the genetic algorithm are repeated until the stop condition is achieved. The stop condition in this algorithm is a predetermined number of generations (iterations).

In this assignment problem the chromosome is represented by string of natural numbers. Such structure of the input data successfully works out in similar issues where the use of zero–one strings significantly hinders the operation of the genetic algorithm, e.g. in the travelling salesman problem[11]. The genetic algorithm does not work directly on the decision variables of the function of criterion but on the encoded forms of these variables. In order to encode the variables of the function of criterion in an appropriate structure and create the chromosome as a representative of the admissible solution the problem of allocation must be defined as the appointment of a suitable permutation of tasks and the base so

[10] Bräysy O., Gendreau M., *Vehicle Routing Problem with Time Windows, Part II: Metaheuristics*, Transportation Science, Institute for Operations Research and the Management Sciences (INFORMS), Volume 39 Issue 1, USA 2005, pp. 119-139.
Goldberg D.E., *Algorytmy genetyczne i ich zastosowanie*, Wydawnictwo Naukowo-Techniczne, Warszawa 1995. Michalewicz Z., *Algorytmy genetyczne + struktury danych = programy ewolucyjne*, Wydawnictwo Naukowo-Techniczne, Warszawa 1996.

[11] Goldberg D.E., Lingle R., *Alleles, Loci, and the TSP*, Proceedings of the First International Conference on Genetic Algorithms, Lawrence Erlbaum Associates, Hillsdale, NJ 1985, pp. 154-159.

that their location will generate the minimum value of the function of criterion, which in our case is the minimum length of the route.

The task of the genetic algorithm is to find the best set of tasks and the base by optimizing the function of adaptation.

The structure of data suitable for processing by the genetic algorithm can be defined as a string consisting of the realized $i-$ tasks and the base.

The total length of the chromosome is $2i+1$ genes. This length of the chromosome is imposed by the situation in which a single vehicle performs only one task and returns to the base (Fig.2.a). In order to realize that situation the base must be encoded in a few genes (Fig.2.b). Each chromosome is a representative of the minimum route. We must remember that the first and last gen of the chromosome is encoded as the gen of the base because the base is always the starting and ending point of the route of each vehicle.

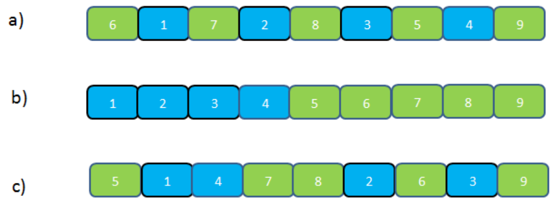


Fig. 2. The structure of chromosome
Source: compiled by author

The minimum route in the chromosome we can interpreted as (Fig.2.c): the vehicle leaves the base (5) and goes to the task (1), leaves the task (1) and goes to the task (4), leaves the task (4) and goes to the base (one base encoded in genes 7,8) leaves the base and goes to the task (2), leaves the task (2) and goes to the base (6), leaves the base (6) and goes to the task (3), leaves the task (3) and goes to the base. The one base is encoded in genes 5,7,8,6,9. There are three elementary routes in the chromosome in Fig.2.c, i.e. 5-1-4-7, 8-2-6, 6-3-9. The individual route for the vehicle can be presented as: 5-1-4-8-2-6, or 8-2-6-3-9, or 5-1-4-6-3-9 etc.

At this moment the length of the individual route depends on the time limits according to the assumption of the second stage of the presented method.

The principle of the operation of the genetic algorithms is to search for maximum value. In order to find the minimum value of chromosomes the function of adaptation for each chromosome takes the form:

$$F_l^{'} = C_{\max} - F_l \qquad (6)$$

where:

$C_{\max}$ – a value larger than the real value of the chromosome,

F_l – the real value of the function of adaptation for l -this chromosome which is equal to the value of the function of the criterion (1).

The maximum value of the function of adaptation $F_l^{'}$ is equal to the minimum value of the function of the criterion.

The selection process consists of the following stages: the calculation of the function of adaptation for a single chromosome, the calculation of the function of adaptation for a total population, the calculation of the probability of the selection l - this chromosome, the calculation of the distribution l - this chromosome. Choosing the chromosome for the next generation basis on the random selection of the number r from the range of [0,1]. We choose l - this chromosome with the value of distribution q_l while the relationship $q_{l-1} < r \leq q_l$ is fulfilled.

To carry out the process of the crossover the PMX crossover was used. The PMX crossover randomly combines two chromosomes in pairs, randomly selects the two points of the crossover and exchanges genes in chromosomes between the points of the crossover.

In the process of the crossover genes from one chromosome are assigned to genes from the other. The crossover occurs with the probability p^k. The PMX crossover is shown in Fig. 3.

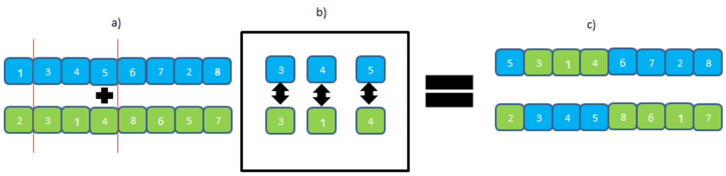


Fig. 3. The PMX a) before the crossover b) a swap of genes c) after the crossover
Source: compiled by author.

A mutation is to swap the places of two randomly selected genes. The mutation occurs with the probability p^m. The principle of the mutation is shown in Fig. 4.

Fig. 4. The use of a mutation operator
Source: compiled by author.

The inversion randomly selects two points in the chromosome. These points create the string which needs to be reversed. The inversion occurs with the probability p^{in} .The principle of the inversion is shown in Fig. 5.

Fig. 5. The principle of the inversion
Source: compiled by author.

Verification of the method

The most important stage of the presented method is the stage of designating the minimum route implementing all the tasks (the first stage of the method) because the individual routes of vehicles are designated on the basis of this route. The process of the verification of the method was conducted at the stage of designating the minimum route, because this stage of the method demands using the optimization tool (genetic algorithm) and decides about further results.

The method was verified using the programming language C#. Verification of the method relies on comparing the results of the genetic algorithm with the results of the random search algorithm. The random search algorithm generates 2000 routes and the best result is compared with the genetic algorithm. The method was verified for 30 tasks, the distances between the tasks and the base are known. The genetic algorithm was started 50 times so the result is the average value of all the starts. The number of iterations is equal to 200, the number of chromosome in the population is equal to 100. The combination of parameters is shown in Tab.1, the parameters take the values:

$$p^k = 0{,}2;0{,}4;0{,}6;08;1,\ \ p^m = 0{,}01;0{,}03;0{,}05,\ \ p^{in} = 0;0{,}4;0{,}6;0{,}8;1.$$

Verification of the method consists of two stages. In the first stage the best parameters of the algorithm are designated. 75 combinations of these parameters were checked and the best combination of parameters, where the algorithm gave the best solution, was found. The influences of the parameters on the quality of the results are shown in Fig. 6, Fig. 7.

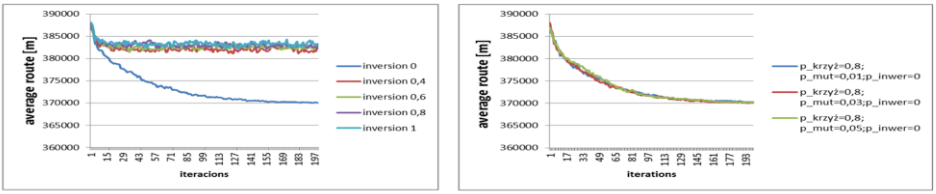
After analysing the results of the genetic algorithm we have come to the conclusion that the inversion in each case does not improve the quality of the solution. The increase of probability of the crossover improves the quality of the solution. For 200 iteration and for $p^k = 1$ we cannot observe the convergence of the algorithm to a specified value, so the number of iterations increased to 400. After changing of the number of iterations the algorithm gave the best solution. The mutation has a minimum influence on the quality of the results. The best result was designated for the parameters (for 400 iterations): $p^k = 1$, $p^m = 0{,}03$, $p^{in} = 0$.

In the second stage the algorithm was started 50 times with these parameters and the average result was compared with the average result of the random search algorithm. The result is shown in Fig. 8.

Tab. 1. The combination of parameters in the genetic algorithm

Lp.	p^k	p^m	p^{in}	Lp.	p^k	p^m	p^{in}	Lp.	p^k	p^m	p^{in}
1	0,2	0,01	0	26	0,4	0,05	0	51	0,8	0,03	0
2	0,2	0,01	0,4	27	0,4	0,05	0,4	52	0,8	0,03	0,4
3	0,2	0,01	0,6	28	0,4	0,05	0,6	53	0,8	0,03	0,6
4	0,2	0,01	0,8	29	0,4	0,05	0,8	54	0,8	0,03	0,8
5	0,2	0,01	1	30	0,4	0,05	1	55	0,8	0,03	1
6	0,2	0,03	0	31	0,6	0,01	0	56	0,8	0,05	0
7	0,2	0,03	0,4	32	0,6	0,01	0,4	57	0,8	0,05	0,4
8	0,2	0,03	0,6	33	0,6	0,01	0,6	58	0,8	0,05	0,6
9	0,2	0,03	0,8	34	0,6	0,01	0,8	59	0,8	0,05	0,8
10	0,2	0,03	1	35	0,6	0,01	1	60	0,8	0,05	1
11	0,2	0,05	0	36	0,6	0,03	0	61	1	0,01	0
12	0,2	0,05	0,4	37	0,6	0,03	0,4	62	1	0,01	0,4
13	0,2	0,05	0,6	38	0,6	0,03	0,6	63	1	0,01	0,6
14	0,2	0,05	0,8	39	0,6	0,03	0,8	64	1	0,01	0,8
15	0,2	0,05	1	40	0,6	0,03	1	65	1	0,01	1
16	0,4	0,01	0	41	0,6	0,05	0	66	1	0,03	0
17	0,4	0,01	0,4	42	0,6	0,05	0,4	67	1	0,03	0,4
18	0,4	0,01	0,6	43	0,6	0,05	0,6	68	1	0,03	0,6
19	0,4	0,01	0,8	44	0,6	0,05	0,8	69	1	0,03	0,8
20	0,4	0,01	1	45	0,6	0,05	1	70	1	0,03	1
21	0,4	0,03	0	46	0,8	0,01	0	71	1	0,05	0
22	0,4	0,03	0,4	47	0,8	0,01	0,4	72	1	0,05	0,4
23	0,4	0,03	0,6	48	0,8	0,01	0,6	73	1	0,05	0,6
24	0,4	0,03	0,8	49	0,8	0,01	0,8	74	1	0,05	0,8
25	0,4	0,03	1	50	0,8	0,01	1	75	1	0,05	1

Source: compiled by author.



a b

Fig. 6. The influence of a) the inversion b) the mutation on the quality of the results
Source: compiled by author.

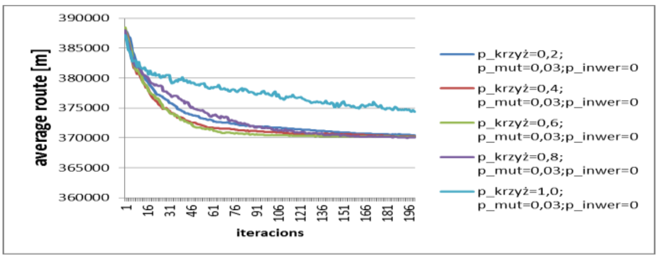


Fig. 7. The influence of the crossover on the quality of the results
Source: compiled by author.

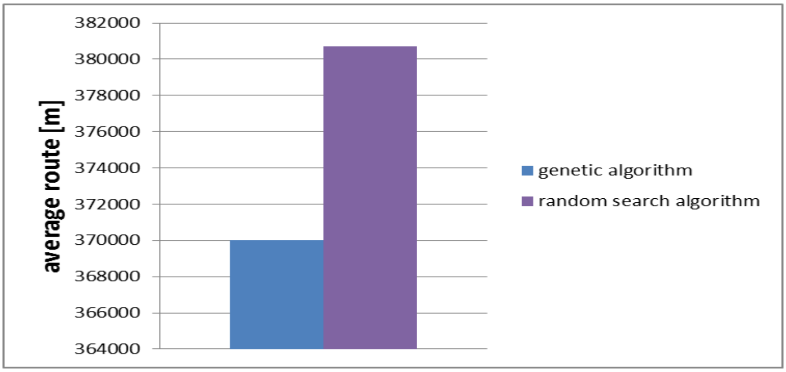


Fig. 8. Comparison of algorithms
Source: compiled by author.

Summary

The purpose of the paper was to define the assignment problem in municipal service companies and to propose a method solving the described problem.

The assignment problem was presented in the context of designating the minimum routes of vehicles therefore the modelling of the process of the assignment is the optimization issue. The proposed method consists of two stages i.e. the stage of designating the minimum route implementing all tasks and the stage of designating individual routes for each vehicle.

The optimization problem of determining the minimum route in the first stage was solved by the genetic algorithm. The presented verification confirmed the effectiveness of the algorithm.

The individual routes of vehicles designated in the second stage are component parts of the minimum route therefore the assignment based on the minimum route is the optimal assignment.

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