Identification of systems reliability

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Abstract. A purpose of the article is discussing the method of technical systems reliability modelling. The most important methods used for constructing realist models of the physical world, i.e. the method of the experimental modelling and the method of the probabilistic modelling, were presented. The particular attention was focussed on the method of the probabilistic modelling allowing for taking into account uncertainty in functioning of systems. An example of using the proposed method to the modelling of the reliability of the air-traffic-control-system was quoted.

Keywords: identification, systems, modelling, reliability of systems

1 Introduction

We are calling the distinguished fragment of real world, of which properties and occurrences happening in it are the subject of the research *the system* [2, 5, 13, 15]. *Technical systems*, understood as the set of technical devices tied together functionally cooperating in performing specific activities, are a special kind of systems.

We are calling creating the mathematical description of the system *the identification* [13, 18, 19, 20, 23]. A *mathematical modelling* is a research method consisting creating mathematical models and using apparatus of mathematics for their analysis. *A mathematical model* is a rough description of the system expressed using apparatus of mathematics reflecting the entire available knowledge about the system.

The theory of reliability [3, 6, 7, 16] is a domain of the research activity being aimed at a cognition and understanding crucial factors affecting the reliability of systems, of especially technical systems. The *reliability* of the technical system is interpreted as its ability for the performance of tasks in named terms and in determined time intervals.

With *reliability identification* we are naming creating the mathematical description of all factors having a significant influence on the reliability of systems.

Out of many paradigms of the identification of systems [13, 18, 19, 20, 23], in the modelling of the reliability of systems of the special significance paradigms are picking up the experimental modelling and the probabilistic modelling.

2 Experimental modelling of systems

The paradigm of the experimental modelling [13, 18] is coming from the concept of empiricism - of philosophical direction in the theory of cognition, leading the human cognition out of sensory, outside or internal experience. This paradigm allows for constructing models of systems on the basis of observations of their input signals and suiting them of output signals.

Let $\mathbb{R}, \mathbb{Z}, \mathbb{N}$ denote, appropriately, a set of real numbers, a set of integral numbers and a set of natural numbers. Let's enter markings: $\mathbb{R}_{(0,\infty)} = (0, \infty) \subset \mathbb{R}$, $\mathbb{R}_{[0,\infty)} = [0, \infty) \subset \mathbb{R}$, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Let's assume that considered system has a m of inputs and a m of outputs. Making this assumption isn't reducing the generality of deliberations, because if $m = \max\{m_u, m_y\}$, where m_u is a number of inputs, m_y is a number of outputs, it is possible to absorb it, that signals on non-existent inputs or outputs always accept nulls.

Let $\mathbf{\mathfrak{U}} = (\mathbf{\mathfrak{U}}, \|\mathbf{u}\|)$, $\mathbf{\mathfrak{U}} = \mathbb{R}^m$, $\|\mathbf{u}\| = \sqrt{\mathbf{u}^T\mathbf{u}}$ denotes a space of the value of the vector input signal of the considered system. Let $\mathbf{\mathfrak{Y}} = (\mathbf{\mathfrak{Y}}, \|\mathbf{y}\|)$, $\mathbf{\mathfrak{Y}} = \mathbb{R}^m$, $\|\mathbf{y}\| = \sqrt{\mathbf{y}^T\mathbf{y}}$ denotes a space of the value of the vector output signal of this system. Let $\mathfrak{T}$ denotes a set of parameters in the time. In issues of the identification, the set $\mathfrak{T}$ is interpreted in the different way. The following cases are distinguished: $\mathfrak{T} = \mathbb{R}$, $\mathfrak{T} = \mathbb{R}_{[0,\infty)}$, $\mathfrak{T} = \mathbb{R}_{(0,\infty)}$, $\mathfrak{T} = \mathbb{Z}$, $\mathfrak{T} = \mathbb{N}$ and $\mathfrak{T} = \mathbb{N}_0$. The mapping $\mathbf{p} : \mathfrak{T} \to \mathbf{\mathfrak{U}}$ we are calling the *input signal* of the system. The mapping $\mathbf{q} : \mathfrak{T} \to \mathbf{\mathfrak{Y}}$ we are calling the *output signal* of the system. If $\mathfrak{T} = \mathbb{R}$, $\mathfrak{T} = \mathbb{R}_{[0,\infty)}$ or $\mathfrak{T} = \mathbb{R}_{(0,\infty)}$ then $\mathbf{p}$ and $\mathbf{q}$ we are calling *continuous mappings*. If $\mathfrak{T} = \mathbb{Z}$, $\mathfrak{T} = \mathbb{N}$ or $\mathfrak{T} = \mathbb{N}_0$, then $\mathbf{p}$ and $\mathbf{q}$ we are calling *discrete mappings*.

The experimental research consists in registering the value $(\mathbf{u}_n, \mathbf{y}_n)$ of couple of input and output signals $(\mathbf{p}(t), \mathbf{q}(t)) \in \mathbf{\mathfrak{U}} \times \mathbf{\mathfrak{Y}}$, where $\mathbf{u}_n = \mathbf{p}(t_n)$, $\mathbf{y}_n = \mathbf{q}(t_n)$, of the considered system with discreet moments $t_n = n\Delta t \in \mathfrak{T}$, for $n = 0, \dots N-1$, where t_0 is a moment of beginning observation of the system, however $\Delta t > 0$ is a time interval between next observations. The set $\{(\mathbf{u}_n, \mathbf{y}_n) \in \mathbf{\mathfrak{U}} \times \mathbf{\mathfrak{Y}}\}_{n=0}^{N-1}$ we are calling the *set of observations* (or briefly: *observations*).

Observations $\{(\mathbf{u}_n, \mathbf{y}_n) \in \mathbf{\mathfrak{U}} \times \mathbf{\mathfrak{Y}}\}_{n=0}^{N-1}$ it is possible to interpret as values of deterministic or random signals. If observations $\{\mathbf{u}_n\}_{n=0}^{N-1}$ and $\{\mathbf{y}_n\}_{n=0}^{N-1}$ are being interpreted as values of deterministic signals, then $\mathbf{p}$ and $\mathbf{q}$ mappings are deterministic functions. If observations $\{\mathbf{u}_n\}_{n=0}^{N-1}$ and $\{\mathbf{y}_n\}_{n=0}^{N-1}$ are being interpreted as values of random signals, then $\mathbf{p}$ and $\mathbf{q}$ mappings are Borel functions.

Methods of the experimental modelling at present are most often used for the identification of structurally and functionally complex systems.

3 Probabilistic modelling of systems

The paradigm of the probabilistic modelling is coming from the concept of uncertainty, according to which no general entitlements or universal regularities determining the development of physical phenomena don't exist. This paradigm allows for constructing realist mathematical models describing in the way moved closer phenomena of the physical world. Probabilistic models are usually expressed using methods of the theory of probability [21], the mathematical statistics and the theory of stochastic processes [1, 3, 16, 22].

Notions of event, elementary event, random events, probabilistic space, random variable and stochastic process are basic universals of the probabilistic modelling.

We are calling the any physical phenomenon associated with the examined system the *event*. Every possible outcome of this phenomenon is an *elementary event*.

Let Ω denotes a *set of elementary events*. Let $\mathcal{F} = \sigma(2^{\Omega})$ denotes a distinguished σ – body of subsets of the set Ω. Let $P : \mathcal{F} \to \mathbb{R}_{[0,\infty)}$ denotes a *probabilistic measure*. We are calling the family of events $\mathcal{F}$, to which the probabilistic measure was assigned, the *family of random events*. Any subset of the set $\mathcal{F}$ we are calling the *random event*.

Definition 1. The triple $(\Omega, \mathcal{F}, P)$ we are calling the *probabilistic space*.

□

Definition 2. Let $(\Omega, \mathcal{F}, P)$ be a probabilistic space. We are saying that the function $x : \Omega \to \mathbb{R}$, determined on the set of elementary events Ω and with values in $\mathbb{R}$, is a *random variable*, if $\{\omega \in \Omega : x(\omega) \in B\} \in \mathcal{F}$, i.e. if the set $\{\omega \in \Omega : x(\omega) \in B\}$ is a random event for every Borel set B.

□

Definition 3. Let $(\Omega, \mathcal{F}, P)$ be a probabilistic space. Let $\mathfrak{T} = \mathbb{R}$ be a set of parameters in the time. Let $\mathbb{R}^{\mathfrak{T}}$ be a set of all functions of real values determined on the set $\mathfrak{T}$, whereas let $B^{\mathfrak{T}}$ be a Borel family subsets of the set $\mathbb{R}^{\mathfrak{T}}$. We are saying that the function $x : \Omega \to \mathbb{R}^{\mathfrak{T}}$, assigning every elementary event from the Ω set to element of the set $\mathbb{R}^{\mathfrak{T}}$, is a *stochastic process with continuous time* (or briefly: *stochastic process*), if $\{\omega \in \Omega : x(\omega) \in B^{\mathfrak{T}}\} \in \mathcal{F}$, i.e. if the set $\{\omega \in \Omega : x(\omega) \in B^{\mathfrak{T}}\}$ is a random event for every Borel set $B^{\mathfrak{T}}$.

□

4 The reliability model of the system

The *reliability model of the system* appoints the triple:

$$(\overline{\mathbb{S}}, \overline{\mathbb{X}}, \overline{\mathbb{D}}), \tag{1}$$

where: $\overline{\mathbb{S}}$ denotes a *model of the system reliability structure* (point 4.1), $\overline{\mathbb{X}}$ denotes a *model of the time evolution of the system reliability states* (point 4.2), however $\overline{\mathbb{D}}$ is a *model of the time evolution of passages of the system* (point 4.3).

4.1 The model of the reliability structure of the system

In theory and engineering of the reliability [3, 6, 7, 16] oneself dynamic and static reliability structure of systems are considering. We are saying that the system reliability structure is *dynamic*, if the cooperation of elements is changing in the time as a result of their damages or changes of the configuration. Otherwise we are saying that the system reliability structure is *static*.
The *model of the dynamic reliability structure of the system* appoints the triple:

$$\overline{\mathbb{S}} \stackrel{\text{def}}{=} (\mathfrak{T}, \mathfrak{E}, \mathfrak{R}), \tag{2}$$

where $\mathfrak{E} = \{e_1, \dots, e_K\}$ is a set of elements, however $\mathfrak{R} \subset \mathfrak{T} \times \mathfrak{E} \times \mathfrak{E}$ is a ternary relation.
The *model of the static reliability structure of the system* appoints the ordered pair:

$$\overline{\mathbb{S}} \stackrel{\text{def}}{=} (\mathfrak{E}, \mathfrak{R}), \tag{3}$$

where $\mathfrak{E} = \{e_1, \dots, e_K\}$ is a set of elements, however $\mathfrak{R} \subseteq \mathfrak{E} \times \mathfrak{E}$ is a binary relation.

Let $\mathfrak{K} \stackrel{\text{def}}{=} \{1, \dots, K\}$ be a set of ID badges of all elements of the reliability structure $\overline{\mathbb{S}}$ (1).

4.2 The model of the time evolution of the system reliability states

With *reliability state* we are naming the smallest numerically set of linearly independent quantities permitting the ability of the system the ambiguous evaluation for the performance of tasks in named terms and in the determined time interval.

The model of the time evolution of the system reliability states appoints the ordered pair:

$$\overline{\mathbb{X}} \stackrel{\text{def}}{=} (\{\mathbb{X}_k\}_{k \in \mathcal{K}}, \mathbb{X}), \tag{4}$$

where:

- $\mathbb{X}_k$ is a *model of the time evolution of reliability states of the element* $e_k \in \mathfrak{E}$;
- $\mathbb{X}$ is a *model of the time evolution of reliability states of the system as a whole*.

The model $\mathbb{X}_k$ (4) appoints the triple:

$$\mathbb{X}_k \stackrel{\text{def}}{=} (\mathfrak{T}, \mathfrak{X}_k, \mathrm{g}_k), \tag{5}$$

where:

- $\mathfrak{X}_k$ is a *set of reliability states of the element* e_k;
- $\mathrm{g}_k : \mathfrak{X}_k \times \mathfrak{T} \to \mathfrak{X}_k$ is a *function of the reliability state transition of the element* e_k.

The model $\mathbb{X}$ (4) appoints the triple:

$$\mathbb{X} \stackrel{\text{def}}{=} (\mathfrak{T}, \mathfrak{X}, \mathrm{g}), \tag{6}$$

where:

- $\mathfrak{X}$ is a *set of reliability states of the system as a whole*;
- g is a *structural function*. It is worthwhile underlining, that if $\overline{\mathbb{S}}$ (1) is a static structure (3), then $\mathrm{g} : \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_K \to \mathfrak{X}$ is a set function, which it is possible to express in the form of sums and products of the random events consisting in changes of reliability states of elements $\{e_k \in \mathfrak{E}\}_{k \in \mathfrak{K}}$. If $\overline{\mathbb{S}}$ is a dynamic structure (2), then $\mathrm{g} : \mathfrak{X}_1 \times \ldots \times \mathfrak{X}_K \times \mathfrak{T} \to \mathfrak{X}$ it isn't possible to present in the form of a set function and it is usually determined in the algorithmic form as the sequence of random events.

4.3 The model of the time evolution of passages of the system

The model of the time evolution of passages of the system appoints the ordered pair:

$$\overline{\mathbb{D}} \overset{\text{def}}{=} (\{\mathbb{D}_k\}_{k \in \mathfrak{K}}, \mathbb{D}), \tag{7}$$

where:

- $\mathbb{D}_k$ is a *model of the time evolution of passages of the element* $e_k \in \mathfrak{E}$;
- $\mathbb{D}$ is a *model of the time evolution of passages of the system as a whole*.

The model $\mathbb{D}_k$ (7) appoints the triple:

$$\mathbb{D}_k \overset{\text{def}}{=} (\mathfrak{T}, \mathfrak{X}_k, \mathrm{f}_k), \tag{8}$$

where $\mathrm{f}_k : \mathfrak{X}_k \times \mathfrak{T} \to \mathfrak{T}$ is a *function of the time evolution of passages of the element* $e_k \in \mathfrak{E}$. The form of this function explicitly isn't usually known. It is assumed that, for the established state $x \in \mathfrak{X}_k$, it is possible to observation of the value $u_{k_{x_i}} = \mathrm{f}_k(x, t_i)$ of this function in the moment t_i, where t_i is a moment of the reliability state transition of the element e_k, for $i = 0,1, \ldots$. The value $u_{k_{x_i}} \in \mathfrak{T}$, where $\mathfrak{T} = \mathbb{R}_{(0,\infty)}$, is being interpreted as a length of the time interval $[t_i, t_{i+1}) \subset \mathfrak{T}$ of staying the element e_k in the state $x \in \mathfrak{X}_k$.

The model $\mathbb{D}$ (7) appoints the triple:

$$\overline{\mathbb{D}} \overset{\text{def}}{=} (\mathfrak{T}, \mathfrak{X}, \mathrm{f}), \tag{9}$$

where $\mathrm{f} : \mathfrak{X} \times \mathfrak{T} \to \mathfrak{T}$ is a *function of the time evolution of passages of the system as a whole*. The form of this function explicitly isn't usually known. Let $u_{x_j} = \mathrm{f}(x, t_j)$ denotes a value of the function f in the moment t_j, where t_j is a moment of the reliability state transition of the system as a whole. The value $u_{x_j} \in \mathfrak{T}$, where $\mathfrak{T} = \mathbb{R}_{(0,\infty)}$, is being interpreted as a length of the time interval $[t_j, t_{j+1}) \subset \mathfrak{T}$ of staying the system as a whole in the state $x \in \mathfrak{X}$. Direct observing values $\{u_{x_j}\}_{j \geq 0}$ isn't usually possible. In practice values of the function f are being generated by way of the stochastic simulation [8, 10].

5 Reliability states

In the theory of reliability are being considered bi- and multi-state reliability models.

Bi-state reliability models. Bi-state reliability models [3, 6, 7, 14, 16] enable the binary quality assessment of tasks performed by the system. In this model only these states are being considered, in which system either fully it is performing its tasks or isn't carrying them out at all. The bi-state model often abides by the assessment of contemporary critical mission computer systems, because these systems in the very nature of things are being assessed only in categories able or disable for the execution of their tasks. In analysis of that kind of systems indirect evaluations aren't being used, because after all these systems were being designed for so that in the constant way they perform all their tasks. The disable of critical mission systems can become a cause of many grave perturbations in functioning of many important aspects of the social life, e.g. of air communication.

Multi-state reliability models. Multi-state reliability models [6, 12] are most often applied to the qualitative assessment of the effectiveness of systems functioning. Tele-transmission systems are an example of multi-state technical systems. These systems are particularly susceptible to outside disruptions of different kind which influence the quality of transmission of signals in the significant way. Let us notice that with reference to that kind of systems stating about their disable isn't tantamount to stating about damaging them.

In some works from the range of the theory of the system reliability [12] attempts of analysis of the reliability of multi-state complex systems are being made, but these results not always are satisfactory. It results mainly from the problems of the computational nature associated with the soaring number of states one should use which to the appropriate description of the process of changes over time of reliability of the system.

Summary. Summing up, it is possible to state that with reference to the outstanding majority of systems of especially the ones which are assembled from electronic elements, the bi-state model is sufficient for conducting their reliability analysis.

Statistical reliability analysis of systems it is possible to conduct based on the statistical material being a result of information reliability examinations.

6 Information reliability examinations

The purpose of information reliability examinations is gathering statistical data about lengths of time intervals $[t_i, t_{i+1}) \subset \mathfrak{T}$ of staying elements $e_k \in \mathfrak{E}$ in states $x \in \mathfrak{X}_k$, for $i = 0,1, \ldots$, $k \in \mathfrak{K}$.

Let's assume that $M \in \mathbb{N}$ systems was provided with examinations and that systems are homogeneous with respect to the reliability. Let $\mathfrak{M} \stackrel{\text{def}}{=} \{1, \ldots, M\}$ be a set of ID badges of these systems. Let $\overline{\mathbb{S}}_m \stackrel{\text{def}}{=} (\mathfrak{E}_m, \mathfrak{R}_m)$, $\mathfrak{E}_m = \{e_1(m), \ldots, e_K(m)\}$, $\mathfrak{R}_m \subset \mathfrak{E}_m \times \mathfrak{E}_m$, be a reliability structure of the m. system, in addition $\overline{\mathbb{S}}_1 = \cdots = \overline{\mathbb{S}}_M = \overline{\mathbb{S}}$, where $\overline{\mathbb{S}}$ (1).

Reliability examinations of systems are being led according to plans:

$$\mathbb{P}_{k_x} = \left(M, R, I_{k_x}\right), \qquad x \in \mathfrak{X}_k,\ k \in \mathfrak{K},$$

which meaning that M systems is subject to examinations; elements $\{e_k(m) \in \mathfrak{E}_m\}_{k\in\mathfrak{K},m\in\mathfrak{M}}$ damaged in the course of the examination are being repaired; the examination is ending after the registration a length of the time interval $\left[t_{I_{k_x}-1}, t_{I_{k_x}}\right) \subset \mathfrak{T}$ of staying the element $e_k(m) \in \mathfrak{E}_m$ in the state $x \in \mathfrak{X}_k$, for $m \in \mathfrak{M}$.

Let

$$\boldsymbol{\mathcal{D}}_{k_x} \stackrel{\text{def}}{=} \left\{D_{k_{x_0}}, D_{k_{x_1}}, \dots, D_{k_{x_{I_{k_x}-1}}}\right\}, \tag{10}$$

be an experiment consisted of attempts $\left\{D_{k_{x_i}}\right\}_{i=0}^{I_{k_x}-1}$. Let $\mathfrak{I}_{k_x} = \left\{0, \dots, I_{k_x} - 1\right\}$ be a set of ID badges of these attempts. Every attempt $D_{k_{x_i}}$ consists in the registration of lengths of time intervals $[t_i, t_{i+1}) \subset \mathfrak{T}$ of staying elements $\{e_k(m) \in \mathfrak{E}_k\}_{m\in\mathfrak{M}}$ in the state $x \in \mathfrak{X}_k$.

The probabilistic space:

$$\left(\Omega_{k_{x_i}}, \mathcal{F}_{k_{x_i}}, P_{k_{x_i}}\right), \qquad i \in \mathfrak{I}_{k_x}, \tag{11}$$

we are calling the *probabilistic model of the attempt* $D_{k_{x_i}}$, what we are filling with writing in the form $D_{k_{x_i}} = \left(\Omega_{k_{x_i}}, \mathcal{F}_{k_{x_i}}, P_{k_{x_i}}\right)$, where:

- $\Omega_{k_{x_i}} = \left\{\omega_{u_{k_{x_i}}(m)} : u_{k_{x_i}}(m) \in \mathbb{R}_{(0,\infty)}\right\}_{m\in\mathfrak{M}}$ is a set of elementary events, where $\omega_{u_{k_{x_i}}(m)}$ is an elementary event meaning that the length of the time interval $[t_i, t_{i+1}) \subset \mathfrak{T}$ of staying the element $e_k(m) \in \mathfrak{E}_m$ in the state $x \in \mathfrak{X}_k$ is taking out $u_{k_{x_i}}(m) \in \mathbb{R}_{(0,\infty)}$. It is worthwhile emphasizing that $u_{k_{x_i}}(m)$ is being interpreted as the value of the function f_k (8) for the established reliability state $x \in \mathfrak{X}_k$ in the established moment t_i of the element $e_k(m) \in \mathfrak{E}_m$;
- $\mathcal{F}_{k_{x_i}} = \sigma\left(2^{\Omega_{k_{x_i}}}\right)$ is a family of random events being distinguished σ-body of subsets of the set $\Omega_{k_{x_i}}$;
- $P_{k_{x_i}} : \mathcal{F}_{k_{x_i}} \to \mathbb{R}_{[0,\infty)}$ is a probabilistic measure.

The probabilistic space:

$$\left(\boldsymbol{\Omega}_{k_x}, \boldsymbol{\mathcal{F}}_{k_x}, P_{k_x}\right), \tag{12}$$

we are calling the *probabilistic model of the experiment* $\boldsymbol{\mathcal{D}}_{k_x}$, $\boldsymbol{\mathcal{D}}_{k_x} = \left(\boldsymbol{\Omega}_{k_x}, \boldsymbol{\mathcal{F}}_{k_x}, P_{k_x}\right)$, where:

- $\boldsymbol{\Omega}_{k_x} = \Omega_{k_{x_0}} \times \Omega_{k_{x_1}} \times \dots \times \Omega_{k_{x_{I_{k_x}-1}}}$;
- $\boldsymbol{\mathcal{F}}_{k_x} = \mathcal{F}_{k_{x_0}} \times \mathcal{F}_{k_{x_1}} \times \dots \times \mathcal{F}_{k_{x_{I_{k_x}-1}}}$;
- $P_{k_x} : \boldsymbol{\mathcal{F}}_{k_x} \to \mathbb{R}_{[0,\infty)}$,

Let $\mathfrak{U}_{k_{x_i}} = \left\{u_{k_{x_i}}(1), u_{k_{x_i}}(2), \dots, u_{k_{x_i}}(M)\right\}$ be a set of results of the attempt $D_{k_{x_i}}$. Let $\boldsymbol{\mathfrak{U}}_{k_x} = \left\{\mathfrak{U}_{k_{x_0}}, \mathfrak{U}_{k_{x_1}}, \dots, \mathfrak{U}_{k_{x_{I_{k_x}-1}}}\right\}$ be a set of results of the experiment $\boldsymbol{\mathcal{D}}_{k_x}$.

In probabilistic modelling of systems we assume that results $\left\{u_{k_{x_i}}(m) \in \mathfrak{U}_{k_{x_i}}\right\}_{m\in\mathfrak{M}}$ of the attempt $D_{k_{x_i}}$ are realizations of functions dependent on elementary events, for $i \in \mathfrak{I}_{k_x}$, $x \in \mathfrak{X}_k$, $k \in \mathfrak{K}$.

7 Random variables and random vectors

Random variables are measurable functions determined on sets of elementary events and with values in Euclidean spaces.

The random variable:

$$\mathcal{u}_{k_{x_i}} : \Omega_{k_{x_i}} \to \mathbb{R}_{(0,\infty)}, \tag{13}$$

we are calling the *random model of results from the set* $\mathfrak{U}_{k_{x_i}}$, where $\Omega_{k_{x_i}}$ (11). The element $u_{k_{x_i}}(m) \in \mathfrak{U}_{k_{x_i}}$ of the set $\mathfrak{U}_{k_{x_i}}$ is interpreting as the *realization* of a random variable $\mathcal{u}_{k_{x_i}}$, what we are filling with writing in the form $\mathcal{u}_{k_{x_i}} = u_{k_{x_i}}(m)$, for $m \in \mathfrak{M}$.
The random vector:

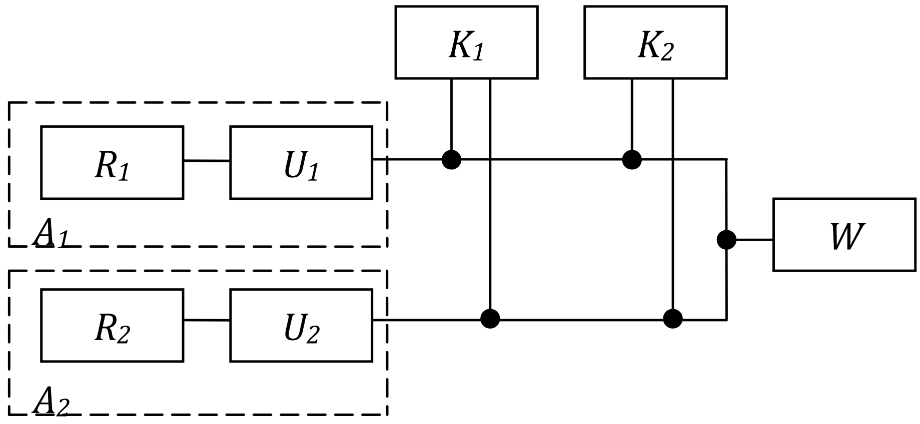
$$\boldsymbol{\mathcal{u}}_{k_x} \stackrel{\text{def}}{=} \left(\mathcal{u}_{k_{x_0}}, \mathcal{u}_{k_{x_1}}, \dots, \mathcal{u}_{k_{x_{I_{k_x}-1}}}\right) : \boldsymbol{\Omega}_{k_x} \to \mathbb{R}^{I_{k_x}}_{(0,\infty)}, \tag{14}$$

we are calling the *random model of results from the set* $\boldsymbol{\mathfrak{U}}_{k_x}$, where $\boldsymbol{\Omega}_{k_x}$ (12). The element $\mathbf{u}_{k_{x_i}} \in \boldsymbol{\mathfrak{U}}_{k_x}$ of the set $\boldsymbol{\mathfrak{U}}_{k_x}$ is interpreting as the *realization* of a random vector $\boldsymbol{\mathcal{u}}_{k_x}$, i.e. $\boldsymbol{\mathcal{u}}_{k_x} = \mathbf{u}_{k_{x_i}}$, for $i \in \mathfrak{I}_{k_x}$.

In the special case, when random variables $\left\{\mathcal{u}_{k_{x_i}}\right\}_{i\in\mathfrak{I}_{k_x}}$ (13) are independent copies of the same random variable $\mathcal{u}_{k_x}$ inducing the probabilistic space $\left(\Omega_{k_x}, \mathcal{F}_{k_x}, P_{k_x}\right)$, the probabilistic space induced by the random vector $\boldsymbol{\mathcal{u}}_{k_x}$ (14) is being marked with symbol $\left(\Omega_{k_x}, \mathcal{F}_{k_x}, P_{k_x}\right)^{I_{k_x}}$. In the theory of reliability is most often considered probabilistic spaces $(\mathbb{R}, \mathcal{F}_{\mathbb{R}}, \{P_\vartheta, \vartheta \in \Theta\})^{I_{k_x}}$, where $\mathcal{F}_{\mathbb{R}}$ is a σ-body of subsets of the set $\mathbb{R}$, however $\{P_\vartheta, \vartheta \in \Theta\}$ is a family of distributions, where Θ is a set of parameters of these distributions.

8 Example

Let's consider the issue of the reliability modelling of the air-traffic-observation-system (picture 1) [24]. Details about the air situation are being delivered from radars R_1 and R_2, through coupling devices U_1 and U_2, for display W. Computers K_1 and K_2 are effecting the data handling about the air situation of observation originating in two channels A_1 and A_2. In case of the breakdown of one of computers, second automatically is adopting objectives of the data handling coming from both channels.



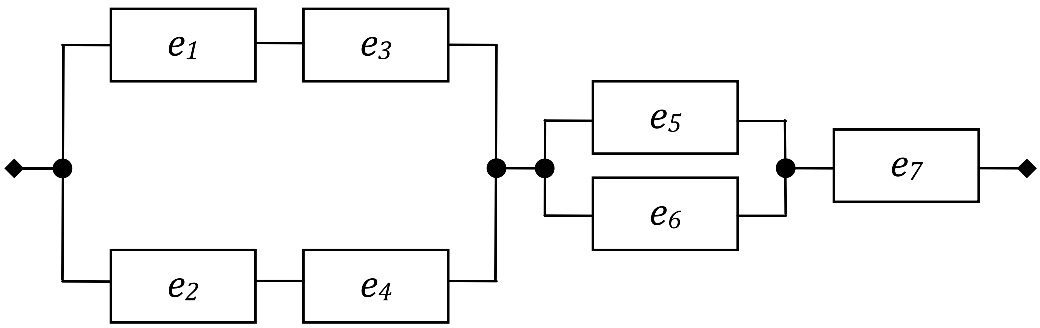
Picture 1. Simplified scheme of the technical structure of the air-traffic-observation-system

Reliability structure. Considering the possibility of the appearance of the breakdown of each elements of the technical structure of the considered system and assuming that the manner of the cooperation of these elements isn't undergoing changes in the time, the model of the reliability structure of the system is static and is assuming the following form

$$\overline{\mathbb{S}} = (\mathfrak{E}, \mathfrak{R}), \tag{15}$$

where:

- $\mathfrak{E} = \{e_1, \dots, e_K\}$ is a set of elements, where: $e_1 \equiv R_1$, $e_2 \equiv R_2$, $e_3 \equiv U_1$, $e_4 \equiv U_2$, $e_5 \equiv K_1$, $e_6 \equiv K_2$, $e_7 \equiv W$, in addition $K = 7$;
- $\mathfrak{R} = \{r_1, \dots, r_8\} \subset \mathfrak{E} \times \mathfrak{E}$ is a binary relation, where: $r_1 = (e_1, e_3)$, $r_2 = (e_2, e_4)$, $r_3 = (e_3, e_5)$, $r_4 = (e_3, e_6)$, $r_5 = (e_4, e_5)$, $r_6 = (e_4, e_6)$, $r_7 = (e_5, e_7)$, $r_8 = (e_6, e_7)$.



Picture 2. Scheme of the static reliability structure of the air-traffic-observation-system

Let $\mathfrak{K} \stackrel{\text{def}}{=} \{1, \dots, K\}$, $K = 7$, be a set of ID badges of all elements of the reliability structure $\overline{\mathbb{S}}$ (15).

Reliability states. Let us accept the bi-state reliability model of the considered system. Let $\mathfrak{X}_k = \{0,1\}$ be a set of reliability states of the element $e_k \in \mathfrak{E}$, in addition $\{0\} \in \mathfrak{X}_k$ means the condition of the disability of this element, however $\{1\} \in \mathfrak{X}_k$ is indicating the condition of its ability, for $k \in \mathfrak{K}$. Let $\mathfrak{X} = \{0,1\}$ be a set of reliability states of the system as a whole, in addition $\{0\} \in \mathfrak{X}$ means the condition of the disability of the system as a whole, however $\{1\} \in \mathfrak{X}$ is indicating the condition of its ability. For the performance of operations on sets $\{\mathfrak{X}_k\}_{k\in\mathfrak{K}}$ and $\mathfrak{X}$ a Boolean algebra is applicable, which is determined by the algebraic structure $\mathbb{B} \stackrel{\text{def}}{=} (0,1,\cup,\cap,\sim)$.

Let $\{t_i \in \mathfrak{T}\}_{i\geq 0}$ be moments of the reliability state transitions of the element $e_k \in \mathfrak{E}$ (15). How it is possible to notice, in moments $\{t_i\}_{i\geq 0}$, the function g_k (5) is assuming the following form

$$x_{k_{i+1}} = \mathrm{g}_k(x_k, t_i) = \sim x_{k_i},$$

where $x_{k_i} = x_k(t_i)$ is a reliability state of the element e_k in the moment t_i, in addition is being accepted that in the moment t_0 the element e_k is able, i.e. $x_k(0) = = x_{k_0} = 1$.

Structural function. For the considered system, the static structural function $\mathrm{g} : \mathfrak{X}_1 \times \dots \times \mathfrak{X}_K \to \mathfrak{X}$ (6) is assuming the following form

$$\begin{aligned} x = \mathrm{g}(x_1, \dots, x_K) &= \big((x_1 \cap x_3) \cup (x_2 \cap x_4)\big) \cap (x_5 \cup x_6) \cap x_7 \\ &= \bigcup_{i=1}^{4} p_i \end{aligned} \tag{16}$$

where $\{p_i\}_{i=1}^{4}$ are ability paths about forms: $p_1 = x_1 \cap x_3 \cap x_5 \cap x_7$, $p_2 = x_1 \cap x_3 \cap x_6 \cap x_7$, $p_3 = x_2 \cap x_4 \cap x_5 \cap x_7$, $p_4 = x_2 \cap x_4 \cap x_6 \cap x_7$, in addition $x \in \mathfrak{X}$, $x_k \in \mathfrak{X}_k$, $k \in \mathfrak{K}$.

Experience. Let's assume that the set $\mathfrak{U}_{k_{x_i}}$ of results of the attempt $D_{k_{x_i}}$ (10) is given, for $i \in \mathfrak{I}_{k_x}$, $x \in \mathfrak{X}_k$, $k \in \mathfrak{K}$.

Random variables. On the basis of the set $\mathfrak{U}_{k_{x_i}}$ it is possible to conduct the verification of the hypothesis concerning the agreement (of fitting) of this set with the chosen theoretical distribution. It is possible to use one of goodness-of-fit-tests for the verification of this hypothesis [1, 11]. To most popular belong: chi-square test of goodness-of-fit, Cramér-von Mises test, Kolmogorov test and Shapiro-Wilk test.

Let's assume that a random variable $\mathcal{u}_{k_{x_i}}$ is a random model of observations from the set $\mathfrak{U}_{k_{x_i}}$ and let $F_{k_{x_i}}$ be a distribution function of this random variable, for $i \in \mathfrak{I}_{k_x}$, $x \in \mathfrak{X}_k$, $k \in \mathfrak{K}$.

Simulation. Since direct observing values of the function $\mathrm{f} : \mathfrak{X} \times \mathfrak{T} \rightarrow \mathfrak{T}$ (9) is impossible, one should conduct a reliability analysis of the considered system with method of the stochastic simulation [8, 10]. An aim of simulation experiment is generating statistical data $\left\{u_{x_j} = \mathrm{f}\left(x, t_j\right)\right\}_{j \geq 0}$ about lengths of time intervals of staying the system as a whole in reliability states $\{x \in \mathfrak{X}_k\}$. It is possible to conduct the simulation with method of discrete events [8] based on the given the structural function g (16) and using pseudorandom number generators [10, 25] about distributions $F_{k_{x_i}}$, for $i \in \mathfrak{I}_{k_x}$, $x \in \mathfrak{X}_k$, $k \in \mathfrak{K}$.

9 Summary

In the paper a method of the reliability modelling of technical systems using paradigms of the experimental modelling and the probabilistic modelling was presented. It is possible to use suggested methodology for drawing up stochastic simulation models enabling to statistical inference about the use process of systems.

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