Simulation modeling of stationary systems reliability

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Abstract: A purpose of the article is describing the methodology of the simulation modeling of systems reliability. At the work is assumed that the process of the use of the studied system is stationary. An algorithm of the simulation applying the technique of discrete-event was presented. An example of using the proposed methodology to the reliability analysis of the air traffic control system was quoted.

Key words: system reliability, modeling, simulation modeling, stochastic simulation

1 Definitions of notions

1. A *system* is a collection of mutually related elements, singled out from surrounding on account of these connections [9].
2. *The theory of reliability* is a field of the research activity being aimed at getting to know and understanding of crucial factors affecting the reliability of systems and other in reality of existing structures [1, 8].
3. A *system reliability* is interpreted as its ability for the performance of tasks in named terms and in determined time intervals.
4. A *reliability state* of the system is called a smallest set of linearly independent elements enabling the ambiguous evaluation of the ability of the system for the performance of tasks in named terms and in the determined time interval. Reliability states are non-measurable.
5. A set of elements and a way of joining them which mapping the influence of the disability of these elements on the disability of the system is called the *system reliability structure*.
6. The smallest set of elements of the system, for which individual measures are well-known, or which gathering data enabling the estimation of these reliability measures is possible for, is called the *element of the system reliability structure*.
7. A process of changes over time of the system reliability states is called the *process of the use of the system*.

2 Introduction

Contemporary technical systems are finding more and more wide applications in many crucial aspects of human activity. These systems more and more are folded technically and technologically. They are in addition more and more big, e.g. contemporary distributed systems are compound of many computers arranged in distant geographically places.

This conditioning causes that with regard to these systems the requirements concerning their reliability are increased. From here a need to develop credible methods of the reliability analysis results. One should regard wrong applying classic methods of reliability, because they require conducting statistical surveys taking entire systems. An aim of these surveys is gathering the statistical material enabling estimation of reliability measures of systems. Since conducting such surveys on contemporary technical systems is difficult, and sometimes give impossible, one should draw it up new reliability analysis methods which don't require performing such examinations. A simulation method is one of promising climbs. A computer simulation is a research method guaranteeing the possibility of effective generating diverse data about theoretically of unrestricted volume. Apart from that it’s a very flexible method. Simulation experiments can be in relatively simple way modified and repeatedly repeated what conducting diverse analytical examinations enables. No other research method is actually ensuring these advantages. Applying this method requires drawing the reliability model and the simulation model of the studied system.

Let’s enter markings: $\mathbb{R}$ - set of real numbers, $\mathbb{R}_{(0,\infty)} = (0,\infty) \subset \mathbb{R}$, $\mathbb{R}_{[0,1]} = [0,1] \subset \mathbb{R}$, $\mathbb{N}$ - set of natural numbers, $\mathfrak{T} = \mathbb{R}$ - set of time parameters.

3 Reliability model of the system

A reliability model of the system is a rough description of time evolution of reliability states of elements and copying the influence of the disability of these elements on the disability of the system.

Definition 1. The reliability model of the system determines the ordered triple:

$$(\overline{\mathbb{S}}, \overline{\mathbb{X}}, \overline{\mathbb{D}}),$$

where: $\overline{\mathbb{S}}$ is a model of the system reliability structure, $\overline{\mathbb{X}}$ is a model of the system reliability states dynamics, $\overline{\mathbb{D}}$ is a model of the system passages dynamics.

□

Definition 2. The model of the system reliability structure determines the ordered pair:

$$\overline{\mathbb{S}} = (\mathfrak{E}, \mathfrak{R}), \tag{1}$$

where $\mathfrak{E} = \{e_1, \dots, e_K\}$ is a set of elements, $\mathfrak{R} \subseteq \mathfrak{E} \times \mathfrak{E}$ is a binary relation.

□

Let $\mathfrak{K} = \{1, \dots, K\}$ be the set of numbers of elements from the collection $\mathfrak{E}$ (1).

Assumption 1. System and elements $e_k \in \mathfrak{E}$, $k \in \mathfrak{K}$, are bi-state with respect to the reliability.

□

Definition 3. Collection $\mathfrak{A}_k = \{0,1\}$ is called the set of reliability states of the element e_k, where $0 \in \mathfrak{A}_k$ is indicating the state of disability of this element, $1 \in \mathfrak{A}_k$ is indicating the state of ability of this element.

□

Definition 4. Collection $\mathfrak{B} = \{0,1\}$ is called the set of reliability states of the system, where $0 \in \mathfrak{B}$ is indicating the state of disability of the system, $1 \in \mathfrak{B}$ is indicating the state of ability of the system.

□

For the performance of operations on sets $\mathfrak{A}_k$, $k \in \mathfrak{K}$, and $\mathfrak{B}$ a Boolean algebra is applicable, which is determined by the algebraic structure $\mathbb{B} = (0,1,\cup,\cap,\sim)$.

Definition 5. The model of the system reliability states dynamics determines the ordered pair:

$$\overline{\mathbb{X}} = (\{\mathbb{X}_k\}_{k \in \mathfrak{K}}, \mathbb{X}),$$

where: $\mathbb{X}_k$ is a model of the time evolution of reliability states of the element e_k, $\mathbb{X}$ is a model of the time evolution of reliability states of the system.

□

Definition 6. The model of the time evolution of reliability states of the element e_k determines the ordered triple:

$$\mathbb{X}_k = (\mathfrak{T}, \mathfrak{A}_k, \mathrm{g}_k), \qquad k \in \mathfrak{K}, \tag{2}$$

where $\mathrm{g}_k : \mathfrak{A}_k \times \mathfrak{T} \to \mathfrak{A}_k$ is a function of reliability states transition of the element e_k.

□

Let $t^{(s)}$, $s = 0,1,...$, be moments of reliability states transitions of elements $e_k \in \mathfrak{E}$, $k \in \mathfrak{K}$. How it is possible to notice the function g_k (2) is accepting in moments $t^{(s)}$, $s = 1,2,...$, the following values:

$$a_k^{(s+1)} = \mathrm{g}_k\left(a_k, t^{(s)}\right) = \sim a_k^{(s)}, \tag{3}$$

where $a_k^{(s)}$ is a reliability state of the element e_k in the moment $t^{(s)}$, and it is accepted that in the moment $t^{(0)}$ the element e_k is able, i.e. $a_k^{(0)} = 1$.

Definition 7. The model of the time evolution of states of the system appoints the ordered quadruple:

$$\mathbb{X} = (\mathfrak{T}, \{\mathfrak{A}_k\}_{k \in \mathfrak{K}}, \mathfrak{B}, \mathrm{g}), \tag{4}$$

where $\mathrm{g} : \mathfrak{A}_1 \times ... \times \mathfrak{A}_K \to \mathfrak{B}$ is a structural function.

□

How it is possible to notice, if $a_k^{(s)}$ is a reliability state of the element e_k in the moment $t^{(s)}$, then the reliability state of the system at that very moment amounts:

$$b^{(s)} = \mathrm{g}\left(\mathbf{a}^{(s)}\right), \tag{5}$$

gdzie $\mathbf{a}^{(s)} = \left[a_1^{(s)}, a_2^{(s)}, \dots, a_K^{(s)}\right]^T$.

Definition 8. The model of dynamics of the system passages appoints the ordered pair:

$$\overline{\mathbb{D}} = (\{\mathbb{D}_k\}_{k\in\mathfrak{K}}, \mathbb{D}),$$

where: $\mathbb{D}_k$ is a model of the time evolution of passages of the element e_k, $\mathbb{D}$ is a model the time evolution of passages of the system.

□

Definition 9. The model of the time evolution of passages of the element e_k determines the ordered triple:

$$\mathbb{D}_k = (\mathfrak{T}, \mathfrak{A}_k, \mathrm{f}_k), \qquad k \in \mathfrak{K}, \tag{6}$$

where $\mathrm{f}_k : \mathfrak{A}_k \times \mathfrak{T} \to \mathfrak{T}$ is a function of passages of the element e_k.

□

It usually to appear that the form of the function f_k (6) explicitly isn't known, but for set states $a \in \mathfrak{A}_k$, values u_{k_a} of the function $\mathrm{f}_{k_a}(t) \equiv \mathrm{f}_k(a, t)$ are well known. In the research on the reliability of systems assumes that these values are interpreted as realization of a random variable $\mathcal{u}_{k_a}\colon \Omega_{k_a} \to \mathbb{R}_{(0,\infty)}$ determined over a probabilistic space $\left(\Omega_{k_a}, \mathcal{F}_{k_a}, \mathrm{P}_{k_a}\right)$, where: $\Omega_{k_a} = \left\{\omega_{u_{k_a}} : u_{k_a} \in \mathbb{R}_{(0,\infty)}\right\}$ is a set of elementary events, $\omega_{u_{k_a}}$ is an elementary event indicating, that observed value of the function f_k, for the set state $a \in \mathfrak{A}_k$, is taking out $u_{k_a} \in \mathbb{R}_{(0,\infty)}$, $\mathcal{F}_{k_a} = \sigma\left(2^{\Omega_{k_a}}\right)$ is a family of random events being a distinguished σ–body of subsets of the set Ω_{k_a}, $\mathrm{P}_{k_a} : \mathcal{F}_{k_a} \to \mathbb{R}_{[0,1]}$ is a probabilistic measure [12].

Definition 10. The model of the time evolution of passages of the system determines the ordered triple:

$$\mathbb{D} = (\mathfrak{T}, \mathfrak{B}, \mathrm{f}), \tag{7}$$

where $\mathrm{f} : \mathfrak{B} \times \mathfrak{T} \to \mathfrak{T}$ is a function of passages of the system.

□

It usually to appear that the form of the function f (7) explicitly isn't known, but for set states $b \in \mathfrak{B}$, values y_b of the function $\mathrm{f}_b(t) \equiv \mathrm{f}(b, t)$ are well known. In the research on the reliability of systems assumes that these values are interpreted as realization of a random variable $\mathcal{y}_b\colon \Omega_b \to \mathbb{R}_{(0,\infty)}$ determined over a probabilistic space $(\Omega_b, \mathcal{F}_b, \mathrm{P}_b)$, where: $\Omega_b = \left\{\omega_{y_b} : y_b \in \mathbb{R}_{(0,\infty)}\right\}$ is a set of elementary events, ω_{y_b} is an elementary event indicating, that an observed value of the function f_k, for the set state $b \in \mathfrak{B}$, is taking out $y_b \in \mathbb{R}_{(0,\infty)}$, $\mathcal{F}_b = \sigma(2^{\Omega_b})$ is a family of random events being a distinguished σ–body of subsets of the set Ω_b, $\mathrm{P}_b : \mathcal{F}_b \to \mathbb{R}_{[0,1]}$ is a probabilistic measure.

In practice individual measures of the reliability of elements e_k, $k \in \mathfrak{K}$, aren't usually known, but it's possible conducting statistical surveys of the reliability of these elements. Gathering statistical material essential to draw the simulation model of the system up is an aim of these surveys.

4 Statistical surveys of the system reliability structure elements

Let's assume that we have a set compound of M, $M \in \mathbb{N}$, homogeneous, in the meaning of the reliability, systems. Let $\mathfrak{M} = \{1, \dots, M\}$ be a set of numbers of these systems. Let's assume that a single system was provided with surveys, of which number was randomly chosen from the set $\mathfrak{M}$. An aim of surveys is gathering realizations of random variables $\mathcal{u}_{k_a}$, $a \in \mathfrak{A}_k$, $k \in \mathfrak{K}$.
Statistical survey of the reliability of element e_k will be conducted according to plan

$$\mathbb{U}_k = (r, L), \qquad k \in \mathfrak{K}. \tag{8}$$

The plan $\mathbb{U}_k$ means that the element e_k damaged in the course of examination is being repaired (r symbol) and the examination is ending after the completion of duration, appearing for the L., $L \in \mathbb{N}$, times, restoration of the element e_k.

The plan $\mathbb{U}_k$ is being made in the form of experiences D_{k_a}, from which every is compound of attempts $D_{k_a}(1), \dots, D_{k_a}(L)$, $a \in \mathfrak{A}_k, k \in \mathfrak{K}$. Let's $\mathfrak{L} = \{1, \dots, L\}$ be a set of numbers of these attempts. Every attempt $D_{k_a}(l)$ consists in observation of the length $u_{k_a}^{(l)} = t_{k_a}^{(1)}(l) - t_{k_a}^{(0)}(l)$ of the time interval $\left[t_{k_a}^{(0)}(l), t_{k_a}^{(1)}(l)\right) \subset \mathfrak{T}$ of staying the element e_k, for the l., $l \in \mathfrak{L}$,times, in the state $a \in \mathfrak{A}_k$. Vector $\mathbf{u}_{k_a} = \left[u_{k_a}^{(1)}, u_{k_a}^{(2)}, \dots, u_{k_a}^{(L)}\right]^T \in \mathbb{R}_{(0,\infty)}^{L\times 1}$ is called the result of the experience D_{k_a} (or briefly: the sample). Observations $\left\{u_{k_a}^{(l)} \in \mathbf{u}_{k_a}\right\}_{l\in\mathfrak{L}}$ are interpreted as realizations of the random variable $\mathcal{u}_{k_a}$. Let $\boldsymbol{\mathcal{u}}_k = \left[\mathcal{u}_{k_0}, \mathcal{u}_{k_1}\right]^T$ be a vector random variable. Matrix $\mathbf{U}_k = \left[\mathbf{u}_{k_0}, \mathbf{u}_{k_1}\right]^T \in \mathbb{R}_{(0,\infty)}^{2\times L}$ is called the result of the plan $\mathbb{U}_k$. Observations $\left\{\left(u_{k_0}^{(l)}, u_{k_1}^{(l)}\right)^T \in \mathbf{U}_k\right\}_{l\in\mathfrak{L}}$ are interpreted as realizations of the random variable $\boldsymbol{\mathcal{u}}_k$.

5 Simulation model of the use process of the system

A computer simulation of the use process of the system is a method of inference about the system reliability on the basis of observations generated by a computer program simulating this process [2, 7]. This type of a computer simulation is being led with discrete-event technique [3, 13]. Let $t \in \mathcal{T}$ be the system time. Sequence of moments $t^{(s)}$, $s = 0,1, \dots$, is called moments of discrete-event incidents, where $t^{(0)}$ is a moment of beginning of the simulation, and $t^{(s)}$ is called the current moment. Events are generated by changes of reliability states of elements e_k, $k \in \mathcal{K}$. Let $a_k^{(s)} \in \mathcal{A}_k$ be a reliability state of the element e_k in the moment $t^{(s)}$ (or: the current state of the element e_k). In moments $t^{(s)}$, $s = 1,2, \ldots$, a current state $a_k^{(s)}$ of the element e_k is settling accounts from the formula (3). Let $b^{(s)} \in \mathfrak{B}$ be a reliability state of the system in the moment $t^{(s)}$ (or the current state of the system). In moments $t^{(s)}$, $s = 1,2, \ldots$, a current state of the system $b^{(s)}$ is settling accounts from the formula (5). It is assumed, that in the moment $t^{(0)}$ all elements are able, i.e. $a_k^{(0)} = 1$, for $k \in \mathcal{K}$. It results from here, that in the moment $t^{(0)}$ the system is able, i.e. $b^{(0)} = 1$. Let $\widehat{\mathcal{u}}_{k_a}$ be a random variable determined by the distribution function $\hat{\mathrm{F}}_{k_a}(t; \mathbf{u}_{k_a}) = P(\widehat{\mathcal{u}}_{k_a} \le t)$ estimated based on the random sample $\mathbf{u}_{k_a}$. In the aim of simplification of notations, let's introduce a symbol $\alpha^{(s)}$ indicating the current state of the element e_k, i.e. $\alpha^{(s)} \equiv a_k^{(s)}$.

The technique of discrete-event consists on generating, in moments $t^{(s)}$, $s = 0,1, \ldots$, of numbers $u_{k_{\alpha^{(s)}}} \in \mathbb{R}_{(0,\infty)}$ determining random lengths of time intervals $\left[t^{(s)}, t^{(s)} + u_{k_{\alpha^{(s)}}}\right)$ of staying the element e_k in the state $\alpha^{(s)}$. These numbers are realizations of the random variable $\widehat{\mathcal{u}}_{k_{\alpha^{(s)}}}$. Random variable $\mathcal{r}_{k_{\alpha^{(s)}}}^{(s)} \stackrel{\text{def}}{=} \mathcal{u}_{k_{\alpha^{(s)}}} - t^{(s-1)}$ is called the remaining time of staying the element e_k in the state $\alpha^{(s)}$. Its realization $r_{k_{\alpha^{(s)}}}^{(s)} = u_{k_{\alpha^{(s)}}} - t^{(s-1)}$ is called an observed in the moment $t^{(s)}$ remaining time of staying the element e_k in the state $\alpha^{(s)}$.

If in the moment $t^{(s)}$, $s = 1,2, \ldots$, a state of the system $b^{(s)}$ changed, in the comparison to the state of the system $b^{(p)}$ in the earlier moment $t^{(p)}$, i.e. in the moment $t^{(p)} < t^{(s)}$, it is observing the length $y_{b^{(p)}}(n_{b^{(p)}}) = t^{(s)} - t^{(p)}$ of the time interval $[t^{(p)}, t^{(s)})$ of staying the system, for the $n_{b^{(p)}}$., $n_{b^{(p)}} \in \mathbb{N}$ times, in the state $b^{(p)} \in \mathfrak{B}$.

6 Simulation survey of the system reliability

Simulation survey of the system reliability is being conducted according to plan

$$\mathbb{Y} = (\{\mathbb{U}_k\}_{k \in \mathfrak{K}}, N), \tag{9}$$

meaning that simulation survey is being conducted based on results of plans $\mathbb{U}_k$, $k \in \mathfrak{K}$; the survey is ending after the completion of duration, appearing for the N., $N \in \mathbb{N}$, times, of disability of the system.

The plan $\mathbb{Y}$ is being made in the form of experiences D_b, from which every is compound of attempts $D_b(1), \ldots, D_b(N)$, $b \in \mathfrak{B}$. Let's $\mathcal{N} = \{1, \ldots, N\}$ be a set of numbers of these attempts. Every attempt $D_b(n)$ consists in observation of the length $y_b^{(n)} = t_b^{(1)}(n) - t_b^{(0)}(n)$ of time interval $\left[t_b^{(0)}(n), t_b^{(1)}(n)\right) \subset \mathfrak{T}$ of staying the system, for the n. times, in the state $b \in \mathfrak{B}$. Vector

$\mathbf{y}_b = \left[y_b^{(1)}, y_b^{(2)}, \dots, y_b^{(N)}\right]^T \in \mathbb{R}_{(0,\infty)}^{N\times 1}$ is called the result of the experience D_b (or briefly: the sample). Observations $\left\{y_b^{(n)} \in \mathbf{y}_b\right\}_{n\in\mathcal{N}}$ are interpreted as realizations of a random variable $\mathcal{y}_b$.

Let $\boldsymbol{\mathcal{y}} = [\mathcal{y}_0, \mathcal{y}_1]^T$ be a vector random variable. Matrix $\mathbf{Y}_k = [\mathbf{y}_0, \mathbf{y}_1]^T \in \mathbb{R}_{(0,\infty)}^{2\times N}$ is called the result of the plan $\mathbb{Y}$. Observations $\left\{\left(y_0^{(n)}, y_1^{(n)}\right)^T \in \mathbf{Y}\right\}_{n\in\mathfrak{N}}$ are interpreted as realizations of the random variable $\boldsymbol{\mathcal{y}}$.

In the attachment an algorithm being an implementation of described methodology of simulation examining of the system reliability was presented.

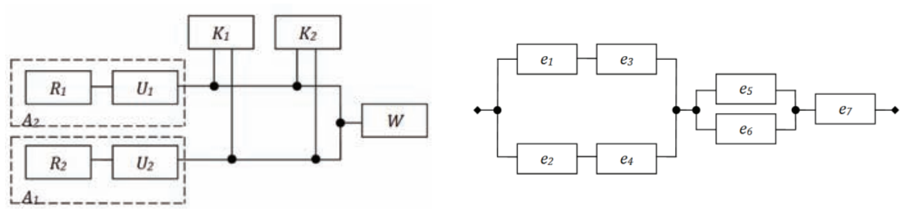
7 Reliability analysis of systems

Analysis of the reliability consists in the estimation of reliability measures of systems based on random sample $\mathbf{y}_b$, $b \in \mathfrak{B}$. For the quantitative evaluation of the reliability of bi-state stationary systems various indicators and functions of the reliability are being used [2, 6, 8]. To most often applied the following empirical measures of the reliability are included:

- sample expected value of the time of the system ability $\hat{\mu}_1(\mathbf{y}_1) = \frac{1}{N}\sum_{n=1}^{N} y_1(n)$;
- sample variance of the time of the system ability $\hat{\sigma}_1^2(\mathbf{y}_1) = \frac{1}{N-1}\sum_{n=1}^{N}[y_1(n) - \hat{\mu}_1(\mathbf{y}_1)]^2$;
- sample expected value of the time of the system disability $\hat{\mu}_0(\mathbf{y}_0) = \frac{1}{N}\sum_{n=1}^{N} y_0(n)$;
- sample variance of the time of the system disability $\hat{\sigma}_0^2(\mathbf{y}_0) = \frac{1}{N-1}\sum_{n=1}^{N}[y_0(n) - \hat{\mu}_0(\mathbf{y}_0)]^2$;
- sample mean availability of the system $\hat{\kappa}(\mathbf{Y}) = \frac{\hat{\mu}_1(\mathbf{y}_1)}{\hat{\mu}_1(\mathbf{y}_1)+\hat{\mu}_0(\mathbf{y}_0)}$;
- sample distribution function $\hat{\mathrm{F}}_1^e(t;\mathbf{y}_1) = \frac{1}{N}\sum_{n=1}^{N} 1\{y_1(n) \le t\}$, which is called the empirical distribution function of the system ability, where $1\{A\}$ is an indication function of the event A;
- sample survival function $\hat{\mathrm{R}}_1^e(t;\mathbf{y}_1) = 1 - \hat{\mathrm{F}}_1^e(t;\mathbf{y}_1)$, which is called the empirical survival function of the system ability;
- sample distribution function $\hat{\mathrm{F}}_0^e(t;\mathbf{y}_0) = \frac{1}{N}\sum_{n=1}^{N} 1\{y_0(n) \le t\}$, which is called the empirical distribution function of the system disability;
- sample survival function $\hat{\mathrm{R}}_0^e(t;\mathbf{y}_0) = 1 - \hat{\mathrm{F}}_0^e(t;\mathbf{y}_0)$, which is called the empirical survival function of the system disability.

8 Example

Let's consider the issue of the reliability analysis of the air-traffic-observation-system (picture 1a) [14, 15]. Details about the air situation are delivered from radars R_1 and R_2, through coupling devices U_1 and U_2, for display W. Computers K_1 and K_2 are effecting the data handling about the air situation of observation originating in two channels A_1 and A_2. In case of the breakdown of one of computers, second automatically is adopting objectives of the data handling coming from both channels.



(a) (b)

Picture 1. Scheme of the technical structure (picture a) and scheme of the reliability structure (picture b) of the air-traffic-observation-system

Reliability structure. Considering the possibility of the appearance of the breakdown of each elements of the technical structure of the considered system and assuming that the manner of the cooperation of these elements isn't undergoing changes in the time, the model of the system reliability structure is static and is assuming the following form

$$\overline{\mathbb{S}} = (\mathfrak{E}, \mathfrak{R}),$$

where:

- $\mathfrak{E} = \{e_1, \dots, e_7\}$ is a set of elements, where: $e_1 \equiv R_1$, $e_2 \equiv R_2$, $e_3 \equiv U_1$, $e_4 \equiv U_2$, $e_5 \equiv K_1$, $e_6 \equiv K_2$, $e_7 \equiv W$;
- $\mathfrak{R} = \{r_1, \dots, r_8\} \subset \mathfrak{E} \times \mathfrak{E}$ is a binary relation, where: $r_1 = (e_1, e_3)$, $r_2 = (e_2, e_4)$, $r_3 = (e_3, e_5)$, $r_4 = (e_3, e_6)$, $r_5 = (e_4, e_5)$, $r_6 = (e_4, e_6)$, $r_7 = (e_5, e_7)$, $r_8 = (e_6, e_7)$.

Structural function. The structural function $\mathrm{g} : \mathfrak{A}_1 \times \dots \times \mathfrak{A}_K \to \mathfrak{B}$ (4) is assuming the following form

$$b = \mathrm{g}(\mathbf{a}) = \bigcup_{i=1}^{4} p_i \tag{10}$$

where: $\mathbf{a} = [a_1, \dots, a_7]^T$, $p_1, \dots, p_4$ are ability paths about forms: $p_1 = a_1 \cap a_3 \cap \cap a_5 \cap a_7$,
$p_2 = a_1 \cap a_3 \cap a_6 \cap a_7$, $p_3 = a_2 \cap a_4 \cap a_5 \cap a_7$, $p_4 = a_2 \cap a_4 \cap a_6 \cap a_7$, $b \in \mathfrak{B}$, $a_k \in \mathfrak{A}_k$, $k \in \mathfrak{K}$.

Reliability surveys of elements $e_k \in \mathfrak{E}$, $k \in \mathfrak{K}$. Let's assume, that as a result of the accomplishment of plans $\mathbb{U}_k = (r, L)$ (8), the following samples were gathered: $\mathbf{u}_{k_a} = \left[u_{k_a}^{(1)}, u_{k_a}^{(2)}, \dots, u_{k_a}^{(L)}\right]^T$, where: $L = 20$, $a \in \mathfrak{A}_k$, $k \in \mathfrak{K}$.

Examining the goodness of fit of samples $\mathbf{u}_{k_a}$, $a \in \mathfrak{A}_k$, $k \in \mathfrak{K}$, with Weibull distributions. For examining the goodness of fit of the sample $\mathbf{u}_{k_a}$ with the Weibull distribution $W(\alpha_{k_a}, \beta_{k_a})$ the Perason-χ^2-test was used. In table 1 results of estimation of the Weibull distribution parameters $\alpha_{k_a}, \beta_{k_a}$ were presented, where $\hat{\alpha}_{k_a}, \hat{\beta}_{k_a}$ are evaluations of parameters $\alpha_{k_a}, \beta_{k_a}$ from samples $\mathbf{u}_{k_a}$. In table 2 results of the test were presented, where: $\hat{t}(\mathbf{u}_{k_a})$ are values of the test statistic, $p - value$ are significance limits of the test.

Table 1. Evaluations $\hat{\alpha}_{k_a}, \hat{\beta}_{k_a}$ of the Weibull distributions parameters $\alpha_{k_a}, \beta_{k_a}$

$k \in \mathfrak{K}$	$a \in \mathfrak{A}_k$	$\hat{\alpha}_{k_a}$	$\hat{\beta}_{k_a}$
1	1	1.71129	287.434
	0	1.59273	2.30877
2	1	1.96253	395.635
	0	1.62945	3.21023
3	1	2.52046	571.206
	0	1.69375	2.65789
4	1	2.18109	575.89
	0	1.62661	3.17491
5	1	2.20431	530.271
	0	1.49464	1.82913
6	1	2.38935	653.281
	0	1.81318	1.51836
7	1	1.76377	163.015
	0	1.95747	7.10472

Table 2. Results of the application of the Perason-χ^2-test for examining the goodness of fit of samples $\mathbf{u}_{k_a}$ with distributions $W(\hat{\alpha}_{k_a}, \hat{\beta}_{k_a})$

$k \in \mathfrak{K}$	$a \in \mathfrak{A}_k$	$\hat{t}(\mathbf{u}_{k_a})$	$p - value$
1	1	10.1	0.120503
	0	3.8	0.70372
2	1	5.2	0.51843
	0	8.0	0.238103
3	1	7.3	0.293992
	0	4.5	0.609339
4	1	5.2	0.51843
	0	5.2	0.51843
5	1	7.3	0.293992
	0	8.0	0.238103
6	1	3.1	0.796195
	0	2.4	0.879487
7	1	8.7	0.191166
	0	4.5	0.609339

It appears from the table 2 that, on the significance level $\alpha = 0.05$, there are no grounds for rejecting null hypotheses stating about the goodness of fit of samples $\mathbf{u}_{k_a}$ with distributions $W(\hat{\alpha}_{k_a}, \hat{\beta}_{k_a})$, $a \in \mathfrak{A}_k$, $k \in \mathfrak{K}$.

Deliverables of simulation experiment. In table 3 model deliverables of simulation experiment conducted according to the plan $\mathbb{Y}$ (9) was presented, where $N = 20$.

Table 3. Model deliverables $\mathbf{y}_b$**,** $b \in \mathfrak{B}$**,** of simulation experiment

$\mathbf{y}_1 =$	[122.968, 43.5962, 104.536, 146.973, 156.167, 67.4747, 80.6407, 34.8864, 256.493, 280.045, 20.8068, 88.3519, 21.013, 122.761, 86.6173, 239.464, 148.43, 27.1738, 59.9506, 68.2416]^T;
$\mathbf{y}_0 =$	[8.07373, 9.25864, 2.3075, 2.37965, 8.68509, 6.11872, 3.89155, 6.9177, 9.91698, 10.4214, 8.50026, 2.99969, 5.98786, 16.3274, 8.46367, 1.63903, 3.50599, 1.09081, 7.61275, 7.18664]^T.

Examining the goodness of fit of samples $\mathbf{y}_b$, $b \in \mathfrak{B}$, **with Weibull distributions**. For examining the goodness of fit of the sample $\mathbf{y}_b$ with the Weibull distribution $W(\alpha_b, \beta_b)$ the Perason-χ^2-test was used. In table 4 results of estimation of the Weibull distribution parameters α_b, β_b were presented, where $\hat{\alpha}_b, \hat{\beta}_b$ are evaluations of parameters α_b, β_b from samples $\mathbf{y}_b$. In table 4 results of the test were presented, where $\hat{t}(\mathbf{y}_b)$ are values of the test statistic.

Table 4. Evaluations $\hat{\alpha}_b, \hat{\beta}_b$ of the Weibull distribution parameters α_b, β_b

$b \in \mathfrak{B}$	$\hat{\alpha}_b$	$\hat{\beta}_b$
1	1.50907	121.165
0	1.87194	7.39298

Table 5. Results of the application of the Perason-χ^2-test for examining the goodness of fit of samples $\mathbf{y}_b$ with distributions $W(\hat{\alpha}_b, \hat{\beta}_b)$

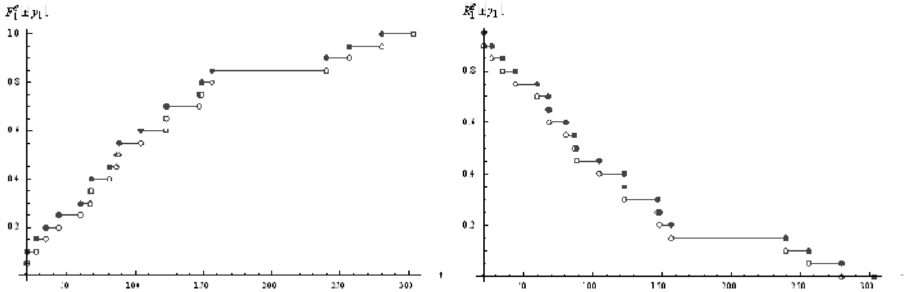
$b \in \mathfrak{B}$	$\hat{t}(\mathbf{y}_b)$	$p - value$
1	2.4	0.879487
0	8.7	0.191166

It appears from the table 5 that, on the significance level $\alpha = 0.05$, there are no grounds for rejecting null hypotheses stating about the goodness of fit of samples $\mathbf{y}_b$ with distributions $W(\hat{\alpha}_b, \hat{\beta}_b)$, $b \in \mathfrak{B}$.

Reliability analysis of the system. In table 6 and in pictures 2 and 3 evaluations of the system reliability measures was described.

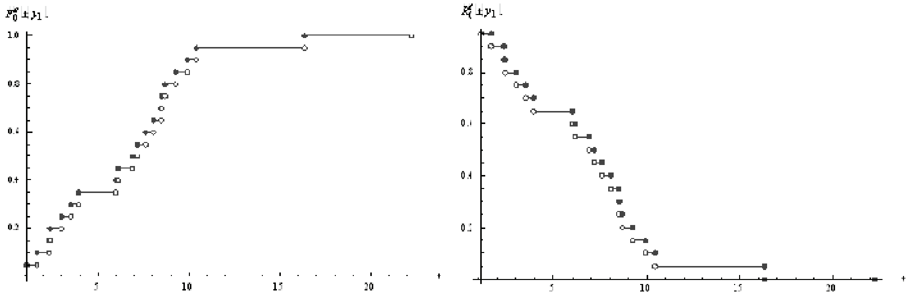
Table 6. Evaluations of the system reliability indicators

$\hat{\mu}_1(\mathbf{y}_1)$	$\hat{\sigma}_1^2(\mathbf{y}_1)$	$\hat{\mu}_0(\mathbf{y}_0)$	$\hat{\sigma}_0^2(\mathbf{y}_0)$	$\hat{\kappa}(\mathbf{Y})$
108.829	5956.08	6.56426	13.8959	0.943114



(a) (b)

Picture 2. Graph of the empirical distribution function of the system ability (a), and graph of the sample survival function of the system ability (b)



(a) (b)

Picture 3. Graph of the empirical distribution function of the system disability (a), and graph of the sample survival function of the system disability (b)

□

9 Summary

At the work a methodology of the simulation modeling of the reliability of stationary systems was described. They showed that a computer simulation was a useful survey method for the reliability analysis of structurally and functionally complex systems.

It is possible to use suggested methodology for solving similar problems, in particular of problems bonded with the evaluation of the effectiveness of technical systems.

Attachment

The following algorithm is an implementation of the methodology of simulation examining of systems reliability.

Algorithm 1.

Input

- length N of the plan $\mathbb{Y}$ (9);
- samples $\mathbf{u}_{k_a}$ being results of plans $\mathbb{U}_k$ (8), for $a \in \mathfrak{A}_k$, $k \in \mathfrak{K}$;

Output

- samples $\mathbf{y}_b = \left[y_b^{(1)}, \dots, y_b^{(N)}\right]^T$ being results of the plan $\mathbb{Y}$ (9), for $b \in \mathfrak{B}$;

I. To perform preliminary calculations

i To conduct estimation of parameters $\alpha_{k_a}, \beta_{k_a}$ of the Weibull distribution $W(\alpha_{k_a}, \beta_{k_a})$ based on the sample $\mathbf{u}_{k_a}$, for $a \in \mathfrak{A}_k$, $k \in \mathfrak{K}$ [5, 10, 11]. Let $\hat{\alpha}_{k_a}, \hat{\beta}_{k_a}$ be evaluations of values of parameters of the Weibull distribution $W(\alpha_{k_a}, \beta_{k_a})$.

ii. To conduct examining the goodness of fit of the sample u_{k_a} with the Weibull distribution $W(\hat{\alpha}_{k_a}, \hat{\beta}_{k_a})$, for $a \in \mathfrak{A}_k$, $k \in \mathfrak{K}$. It is possible to conduct this examination with the help of the Perason-χ^2-test [5, 11]. Let $\hat{\mathrm{F}}_{k_a}(t; \mathbf{u}_{k_a})$ be a distribution function of Weibull distribution $W(\hat{\alpha}_{k_a}, \hat{\beta}_{k_a})$. Let $\hat{\mathrm{u}}_{k_a}$ be a pseudorandom number generator about the distribution determined by the function $\hat{\mathrm{F}}_{k_a}(t; \mathbf{u}_{k_a})$ [4, 16].

II. To establish initial conditions

a. To set the initial value of the step of the simulation process:

$$s = 0.$$

b. To set the initial value of the system time:

$$t^{(0)} = 0.$$

c. To set initial values of reliability states of elements e_k:

$$a_k^{(0)} = 1, \qquad k \in \mathfrak{K}.$$

d. To set initial values $u_{k_1}^{(1)}$ of lengths of time intervals of staying elements e_k in, appearing for the first time, of ability states:

$$u_{k_1}^{(1)} = \hat{\mathrm{u}}_{k_1}, \qquad k \in \mathfrak{K}.$$

e. To set initial values $r_{k_1}^{(0)}$ of observed in the moment $t^{(0)}$ remaining times of staying elements e_k in states $a_k^{(0)}$:

$$r_{k_1}^{(0)} = u_{k_1}^{(1)}, \qquad k \in \mathfrak{K}.$$

f. To set the initial value of the system reliability state:

$$b^{(0)} = 1.$$

g. To set the initial value of the time of staying the system in the current reliability state:

$$y^{(0)} = 0.$$

h. To set the initial values of counters of appearances of the system reliability states:

$$n_b = 1, \qquad b \in \mathfrak{B}.$$

III. To perform calculations

0. Let $\alpha^{(s)}$ means the current state $a_k^{(s)}$ of the element e_k, i.e. $\alpha^{(s)} \equiv a_k^{(s)}$.
1. To calculate the time interval to the incident of the next event:

$$\tau^{(s)} = \min\left\{r_{\alpha^{(s)}}^{(s)} \in \mathbb{R}_{(0,\infty)} : k \in \mathfrak{K}\right\}.$$

2. To bring the value of the system time up to date:

$$t^{(s+1)} = t^{(s)} + \tau^{(s)}.$$

3. To set the number $\hat{k} \in \mathfrak{K}$ of this element of the system reliability structure which the state transition appoints the next event:

$$\hat{k} = \arg\min_k \left\{r_{k_{\alpha^{(s)}}}^{(s)} \in \mathbb{R}_{(0,\infty)} : k \in \mathfrak{K}\right\}.$$

4. To bring up to date values of reliability states of all elements of the system reliability structure for the moment $t^{(s+1)}$:

$$\alpha^{(s+1)} \equiv a_k^{(s+1)} = \begin{cases} \sim a_k^{(s)}, & for\ k = \hat{k}, \\ a_k^{(s)}, & for\ k \neq \hat{k}, \end{cases} \qquad k \in \mathfrak{K}.$$

5. To bring values of variables $r_{k_{\alpha^{(s+1)}}}^{(s+1)}$ up to date:

$$r_{k_{\alpha^{(s+1)}}}^{(s+1)} = \begin{cases} \hat{u}_{k_{\alpha^{(s+1)}}}, & dla\ k = \hat{k}, \\ r_{k_{\alpha^{(s)}}}^{(s)} - \tau^{(s)}, & dla\ k \neq \hat{k}, \end{cases} \qquad k \in \mathfrak{K}.$$

6. To bring the value of variable $y^{(s)}$ up to date:

$$y^{(s+1)} = y^{(s)} + \tau^{(s)}.$$

7. To estimate the state of the system in the moment $t^{(s+1)}$:

$$b^{(s+1)} = \mathrm{g}\left(\mathbf{a}^{(s+1)}\right),$$

gdzie: g (5), $\mathbf{a}^{(s+1)} = \left[a_1^{(s+1)}, \ldots, a_K^{(s+1)}\right]^T$.

8. If $b^{(s+1)} \neq b^{(s)}$, to make the following operations:

$$\mathbf{y}_{b^{(s)}}\left[n_{b^{(s)}}\right] = y^{(s+1)},$$

$$n_{b^{(s)}} \leftarrow n_{b^{(s)}} + 1,$$

$$y^{(s+1)} = 0.$$

9. If $b^{(s)} = 0$ and if $n_{b^{(s)}} > n_Y$, then finish the algorithm, and otherwise to continue calculations from point 10.
10. To bring the value of variable s up to date:

$$s \leftarrow s + 1.$$

11. To continue calculations from point 1.

□

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