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On mapping onto self-organized criticality

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Abstract: In the report we have discussed a few aspects of SOC concept which in general have strongly influence on explicitness of mapping process. SOC idea is based on group of models and/does not seem to give quite clear instructions whether mapped phenomena exhibit SOC or not. To present the problem we have performed a computer simulation in order to investigate the effect of the critical point within the system evolution process without conservation. We have considered that on two-dimensional cellular automata whose rule consists of one or two subrules. The first one, based on Conway's model (or very similar to), has represented the local behavior of transmission processes and has been applied in the experiment synchronously, as a fundamental mode. The second one, called a transport rule, has been applied sequentially. That subrule has described the motion of a fraction of individuals. As a result of comparing models of the various sets of rules for the applied size of lattice, we could find that the modified Conway's model would be merely treated as subcritical.

Keywords. Cellular automata, self-organizing systems, criticality

1 Introduction

The idea of self-organized criticality (SOC) concerns, among other, two terms of mechanism of self-organization and criticality. The aim of the former is focused on the system structure which often appears without explicit pressure or influence from outside the system. The observed change of system behavior results from the interactions among the components. In spite of internal characteristic of constraints between components, it is usually emphasized that global behavior of the system is independent of their physical nature. Self-organized system has, among other, the following typical features: global order, dissipation, instability, multiple equilibria, complexity, criticality. If we add, that the existing dissipation phenomena is not critical, to the concept itself, one can say that this new theory gives results applicable to all other systems characterized by similar network features. The creation process of the new SOC theory will be discussed further.

There are some examples of self-organized systems such as crystallization or different examples of magnetization and Bernard phenomenon or certain chemical reactions.

The second term i.e. criticality is a complex notion regarding phase change, edge of chaos, percolation phenomena and others. One could explain criticality as a point at which the system properties change suddenly but that sounds as another term: the phase change defined similar to the point at which appearance of the system changes suddenly. Then describing criticality is referred to the critical point of the system where a small change can push the system either into chaotic activity or into frozen constancy (e.g. equilibrium). The mechanism through which a complex system tends to maintain on this point (treated as edge of chaos) has been called self-organized criticality by Per Bak et. al. [2]. When we observe system behavior we can find that large adjustments are possible but they are much more rare then small ones. So in that way we formulate behavior of a complex system governed by "the power law".

The concept of self-organized criticality (SOC) is based on observation of near-critical behavior of many natural phenomena. Indeed, the SOC brings those ideas together with underlying explanation for some observed, obvious universal, dynamics of complex systems. SOC would be characterized by structural approach within which we determine, or rather simplify, the nature focused on our analyses on components and their constraints. The fractal description would be a good example here: machines and organizations are designed to be fractal [1]. Both of them are made of parts and on lower levels are made of components or subassemblies and so on. One could expect the distribution of the number of parts of a machine vs. the masses (or volumes) of those parts to be in a form of the inverse power law. But in that case power describes the dimension of fractals [1]. This approach is valid for a self-similar system such as the above-mentioned fractal organizations which are scale-free, what means that they look the same at any scale. The considered power law as a scale-free function would support the thesis that any SOC system is fractal (not necessary as regular, mathematically described fractal but "as heterogenous fractal"). The SOC, according to the definition [1] extended by the fractal definition above, could be treated as ability of a system to evolve in such a way as to approach a critical point and then remain at that point. There are many examples of self-organized criticality which include: sandpiles (sand or rice piles of selected grains), earthquakes, pulsar glitches, solar flares, turbidity layers, people killed in war conflicts, traffic jams, variations in cotton futures, forest fires, landscape formation, river network, mountain ranges, volcanic activity, fluctuations on the stock market, brain functions, spreading of epidemics and so on.

From that point of view SOC seems to link multitude of complex phenomena which we observe in real world to simplistic laws and underlying process (if necessary) hence for many the SOC, like the systems theory that was very popular in the late 60's, is ubiquitous. SOC, when we accept that, may be used to model events as diverse as aforesaid and we adopt that as a new paradigm for the explanation of complex phenomena. Naturally unifying theories has a long history as the recent examples of the string theory, the catastrophe theory and the chaos theory, for instance. The general question is whether the SOC explains us how the nature works. Now we will discuss a few further aspects of the SOC. SOC belongs to far-from-equilibrium class of events. Dissipation phenomena within a self-organized system are expected, although as we mentioned above not critical to the concept itself except for some cases such as the sandpile model (if dissipation events are introduced the sandpile model loses criticality [9]). Some authors claim that non-conservative sandpile model may be SOC [15]. That diversity approach highlights the basic theoretical problems of identifying physically relevant mechanisms for SOC. There are many presented examples of physical events with no apparent conservation law e.g. earthquakes, forest fires, solar flares, but the sandpile model needs conservation [16]. Most striking is treating the sandpile model, at the same time, as a toy model for an earthquake [15]. Further presented models of artificial systems have no analogue for physical phenomena. So such systems can exhibit self-organization criticality in non-conservative mode.

Phase change as a physical term is defined by properties of a system which changes suddenly. An example of a phase change is change of a state of matter from solid to liquid. In the described case critical state has another sense as a term common to thermodynamics. As usual we refer to that in the pressure-temperature phase-space of liquid-gas system. Such a system at the critical point does not exhibit SOC properties (state would be reached and maintained by applying of external forces only), although for example gas bubble distribution follows the power law [17]. Artificial (non-physical) systems can also expose phase changes. However, in case of modeling internal energy of an element of a system, use of that term is more questionable.

The aspects described above may raise doubts that SOC does not seem to be a scientific theory. SOC is based on group of models (e.g. stochastic model, external model) which are "applied" to different domains by reinterpreting the elements of one of the models [18]. In spite of that, SOC is currently developed and very useful idea as a new way of thinking about processes and complexity of the systems.

In this paper we have performed a computer simulation in order to investigate the effect of the critical point within the system evolution process without conservation. It seems that, in spite of more than a decade of intensive studies, the SOC phenomenon is far from being fully understood [12]. That early approach is based on the above-described physical model to explain a vast class of self-similar behavior which can often be found in physical systems. For example Bak's sandpile model was developed as a specific model of self-organizing criticalities.

The way to interpret the time evolution of a complex system is the construction of a model. It is impossible to include all features of a real system in the model so such a simplified description requires more intuition for understanding that. An excellent tool for the described purpose are cellular automata (CA) which support simple models for a number of complex systems. The proper question is whether some models based on CA, especially Conway's model (the Game of Life model), exhibit self-organized criticality or not. There are two groups representing different opinions. The former [4] concludes that a finite-size scaling analysis shows that the Life model is critical while the latter [5] argues that the observed power law behavior is an artifact of the small system sizes, however on larger scales the Life model is subcritical or not critical at all [6]. The aim of this research was to test the two models, the Life model and the similar one, for critical behavior.

2 The model

The cellular automata, as a model of complex systems, contain a large number of identical elements. The interactions between them occur on the local level only. In both models the value of a variable at each site, for given time, depends on the values of variables at the neighboring sites [7]. All sites are updated simultaneously because of the synchronous evolution rule applied. For both the Life model and the similar one (the LifeLike model [8]) the local rule applied synchronously can be presented in the following form:

Life model:

$$\text{rule}(0, \rho) = \begin{cases} 1 & \rho = 3 \\ 0 & \textit{otherwise} \end{cases}$$

$$\text{rule}(1, \rho) = \begin{cases} 1 & 2 \le \rho \le 3 \\ 0 & \textit{otherwise} \end{cases}$$

LifeLike model:

$$\text{rule}(0, \rho) = \begin{cases} 1 & \rho = 3 \quad \textit{or} \quad \rho = 6 \\ 0 & \textit{otherwise} \end{cases}$$

$$\text{rule}(1, \rho) = \begin{cases} 1 & \textit{otherwise} \\ 0 & 2 < \rho < 5 \quad \textit{or} \quad \rho > 5 \end{cases}$$

For the experiment there is applied a set of rules consisting of two subrules. The first one, described above, is a local rule inspired by the Conway's Game of Life or similar one. That rule represents the local behavior of transmission processes. The second one describes the motion of a fraction of live sites. This kind of process is an important aspect of the global evolution models such as an epidemic model [11]. After Boccara [11] and Mansilla [12], we refer to this type of rules as transport rules. Both subrules were enhanced a little for the purposes of a numerical experiment. The subrule governing the Life evolution updates each site with some probability *p*. For the experiment we apply random updating on the level at which the system is nearly asynchronously.

The second subrule describes a mixing process which is responsible for perturbation in the CA system. On the contrary to the CA standard, that subrule is based on looking for free (dead) site processes. The second subrule has an impact on the finally growing number of live sites. That searching process is sequential and can be deterministic or random choice.

The modeling processes are observed at the finite lattice space with the cold boundaries which means that all sites beyond the border are dead.

3 The critical exponent

The typical system is subjected to external factors which have an influence on changing a local state variable. When this variable reaches a critical value we observe in several cases transport process triggering off series of events. That chain of reaction we call an avalanche. The statistic of the avalanches obeys critical scaling in a form of the power law [2][3]:

$$D(t) \sim t^b$$

where $D(t)$ is the number of avalanches of size t and $b < 0$ is a critical exponent – usually non-integer. For the purposes of testing both models described above we are interesting in that frequency distribution for a strong evidence of existing the SOC. The log-log plot of avalanches size against the number of avalanches of that size is a straight line [1] [5].

We should have awareness that subsequent stationary state of a system in whole repeated perturbation-stabilization process does not depend on the initial configuration. When transient as an intermediate behavior exists, some artifacts of the seed may remain for many perturbations after. Figure 3.1 shows the steps evolution of the number of live cells. The result suggests that we could neglect transient period effect (starting from 15% of steps number we have nearly straight, horizontal runs).

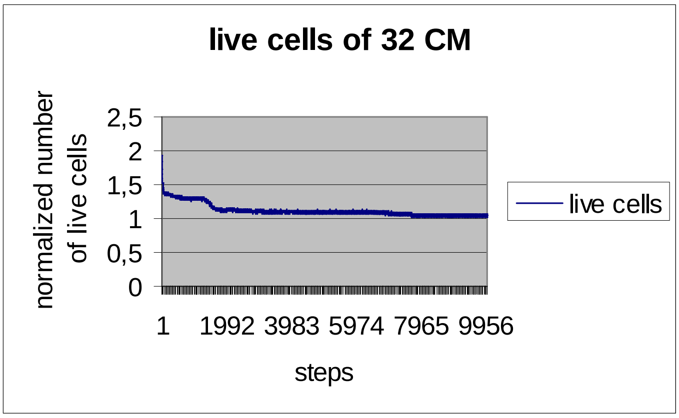

Figure 3.1. Evolution of the number of live cells over perturbations (example of the case of Conway's model for lattice of 32×32)

We look for the SOC behavior in the following runs with the cold boundary conditions employed. All runs are plotted below in Figure 3.2 and Figure 3.3 for four different lattice sizes: 32, 63, 128 and 150. Notice that for not very large avalanches the raw plot in its tail includes sets of sparse points which are separated by values of zero frequency.

In this region the raw plot considerably decays from the power law run, so in order to estimate the best fitting we have to pass over the tail (for our experiment results $t > 190$). However, without applying smoothing techniques we can observe good fitting within the above-mentioned range.

We estimate in this region an average critical exponent for the analyzed size of lattice and we obtain the following values of *b*: for Conway's the Life model: -1,506 +0,063/-0,057 and for the LifeLike model: -1,441 +0,102/-0,04.

The subrule of the local behavior of a transmission process exerts an influence on diversity of values of the average critical exponent between results for the Life and LifeLike models mentioned above. The results obtained can be compared with previous measures of that quantity for the Life model e.g. $b = -1.6$ [3], $b = -1.41$ [4] and $b = -1.175$ [6]. In general it seems to be in compliance with our results but we must emphasize the diversification of conditions on which the cited results were obtained. Bak et. al. [2] explore the lattice with cold boundaries up to the size 150 × 150, Blok [6] analyzes the SOC behavior testing the Conway's model with several boundaries conditions (including two-sided cold boundaries) on the lattice up to 256 × 256, Alstrom [4] tests the lattice up to 1024 × 1024.

That discrepancy points out that we have to identify more factors which have an influence on value of an average critical exponent. In this paper we analyze only one factor. We formulate hypothesis that the critical exponent is integrated with estimation of finite-size effects [6] [9]. To carry out that we construct, after [6], the cumulative distribution of lifetimes $C(t)$ presented in Figure 3.4. The distribution of lifetimes in the finite system due to the lack of a characteristic size and time scales is rewritten as [6][9]:

$$D(t) \sim t^{-(1+b)}$$

where all variables have an identical meaning as presented in the first definition of frequency distribution of avalanches lifetimes. The cumulative distribution obeying a finite-scaling law can also be written in a similar form [6]. The scaling function can be expanded it in the Taylor polynomial. One can truncate expansion at the first order and find that [6] the fitting function can be written as:

$$C(t) = t^{1+b}\, e^{h1\, t/tc}$$

where $C(t)$ – cumulative distribution of lifetimes, $h1$ – an expansion factor in the first order with accepted value: $h1 = -1$, tc – critical lifetime indicating the range over which the power law is valid.

On the basis of the modified cumulative distribution we can find adjustable parameters *b* and *tc* for both models and all sizes of lattice. As a result we find for the Life model the set of average values of parameters: *b* = -0,955 +0,085/-0,055; *tc* = 14,49 +2,8/-1,2 and for the LifeLike model: *b* = -0,769 +0,063/-0,039; *tc* = 12,56 +1,0/-0,8.

Unfortunately, the comparison of these results for the Life model with Blok's ones [6] is not favorable for them but could be a very interesting base for possible explanations and interpretations.

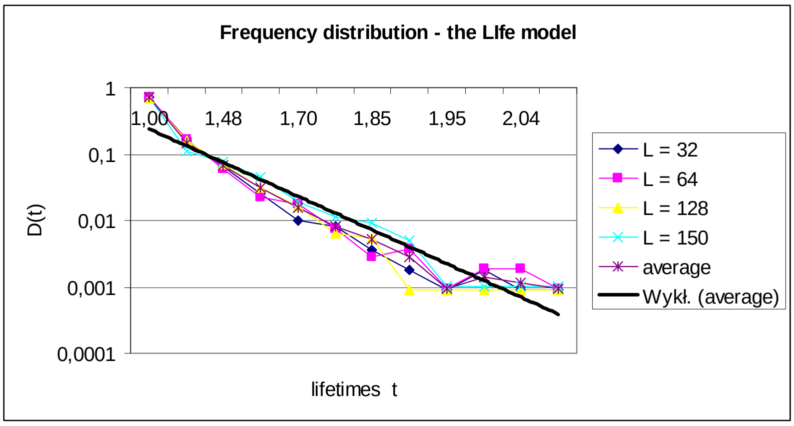

Figure 3.2. The raw frequency distribution of lifetime for the Life model, L × L lattice of four different sizes L: 32, 64, 128 and 150. The Wykł. curve interpolates the average frequency distribution graph

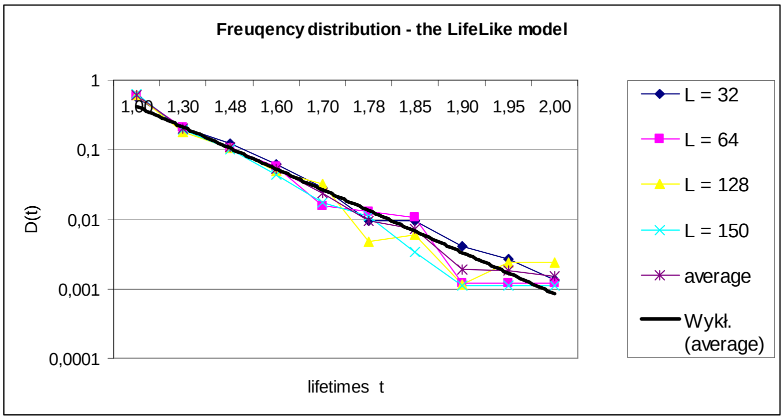

Figure 3.3. The raw frequency distribution of lifetime for the LifeLike model, L × L lattice of four different sizes L: 32, 64, 128 and 150. The Wykł. curve interpolates the average frequency distribution graph

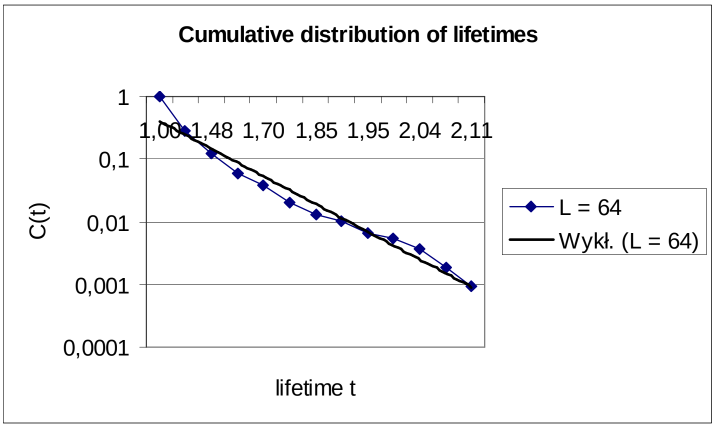

Figure 3.4. Example of the cumulative distribution of lifetimes for the Life model on a cold lattice 64 × 64

4 Conclusions

Idea of self-organized criticality systems is currently developed and seems to have multitude of successful applications. In spite of that SOC does not seem to be a scientific theory. SOC is constructed on a base of a few models and each of applications is reinterpreted by one of them. That creates a problem of identification whether a system exhibits SOC properties or not. A self-organized critical system represents a wide class of open, nonlinear and far from equilibrium complex systems. That kind of systems can spontaneously develop without any even fine tuning. The SOC model based on the Conway's model or a similar one needs enhancement of the standard rules or modification of the boundaries definition because the modeled SOC system by definition has to be open. In this paper we analyze the lattice with all boundaries defined as cold. We test behavior of both models in the determined region so we could not give any conclusions above it.

The first approach as analysis of raw distribution of lifetimes, after ignoring values of a plot tail, suggests, with notice given above, that both the Life and the LifeLike models are subcritical (both are some modifications of the original Conway's model). The values of lifetimes of the analyzed models point out on the existence of strong influence of boundaries effects: for example we have average lifetimes of order 10^2 while Blok [6] has nearly 10^3 (however the next are ignored). The possible explanation of that we can see in interactions of the gliders in case of cold boundaries. The cold boundaries effect integrated with the finite size can be observed through comparing values of an average critical exponent from the raw distribution of lifetimes with its values from the scaling function e.g. for the Life model b = -1,506 for the first distribution and for the last one b = -0,955.

Both models are similar in character but the LifeLike one produces a completely different pattern [8]. One can describe that in a form of sets of 2 × 2 blocks all aligned. This pattern will evolve with the same form adding rare gliders and many different oscillators. The evolution process will strongly decay and we can expect lower values of lifetimes than the original Life model exhibits (e.g. comparing the maximum values of parameters – average lifetime/its frequency: 835,5/97,2 (LifeLike) and 1064,0/168,7 (Life)).

The received values of critical lifetimes *tc* are astonishingly low and point out on a strong affect of edges. The self-organized critically system are inherently based on the physically proper conservation law. The analyzed CA systems are artificial and non-conservative. We remember that the SOC is based on suddenly changing system properties from disconnected to connected state or vice versa. It can be described through change on the level of an elementary particle of the system or the whole system. The first explanation we can connect with the change from solid to liquid as a good example of physical conservative systems. The second one we can see in Thom's catastrophe theory [14] (ignoring many proposed applications), considering CA systems described in this paper etc. This problem is open, considering the class of dissipative and conservative models [9]. When one can find examples of systems, that obey the power law and exhibit no SOC features and on the other hand the same phenomena could be mapped onto SOC model under the conservative law or not, we could claim that SOC is very inspiring idea but development of that is far from the end.

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